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Economic evaluation of proposed uranium mining expenditures under conditions of uncertainty

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by

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I. INTRODUCTION

The purpose of this paper is to demonstrate a methodology for evaluating the economic feasibility of a proposed uranium mining project with uncertain future prices and variable operating costs. If perfect knowledge existed concerning all the determinants of economic feasibility, including prices, investment costs, operating costs, and so on, then determining the Average Annual Rate of Return (AARR) on investment on a cash flow basis is easily accomplished. However, if uncertainty exists with respect to one or more variables, a point estimate of AARR has limited value. But despite the fact that perfect knowledge is unavailable, managers must still make investment decisions based on their expertise, even though their expert opinions often differ. This paper quantifies the differing opinions and presents the underlying collective opinion of a panel of project evaluators as to the risk and profit potential of the proposed mining project.

The procedure will be developed using data solicited from a major oil company pertaining to a proposed uranium mining project in New Mexico. The derivation of AARR using the discounted cash flow method is shown first. After discussing present feasibility evaluation procedures and reviewing the literature on uncertainty, a statistical analysis will be described in detail to analyze the uncertainty five mining experts portrayed in the price and variable operating cost data they provided. Included is a critical evaluation of the method.

The objective of the statistical method is to produce a frequency distribution on AARR. The frequency distribution will estimate the actual probability distribution function on AARR. This information can be of great value to the manager when selecting among several proposed projects.

The procedure will be especially suitable for computer programming. Therefore, this paper will also serve as a foundation for what is commonly called a User-Analyst Report.

The user, or manager, is directed to the Procedure for Gathering the Data, Section V.B., the Results and Interpretation in Section V.D., and the Critical Evaluation, Section V.E.

The economic (computer) analyst is directed to all of Chapter V. The Review of the Literature, Chapter III, might also be of interest to the analyst.

II. DETERMINING AVERAGE ANNUAL RATE OF RETURN

A. Introduction

Given a data set including numerical values for all the determinants of economic feasibility, such as prices, investment, and operating costs schedules and single value data, such as depletion and tax rates, a one point estimate of return on investment can be calculated, which will be called Average Annual Rate of Return or AARR. The discounted cash flow method will be used to calculate AARR. This basic method, or tool, will be used to calculate differing AARR's as different price and operating cost schedule combinations are considered as possible (probable) situations under which the mine may operate. The probability of each different combination of price and cost vectors will be assigned to its respective value of AARR since the remainder of the data is considered determinate. Thus, to construct a frequency distribution on AARR, a method must first be established to calculate AARR.

B. The Discounted Cash Flow Method

The discounted cash flow method of determining return on investment is most easily explained by example.

If one was to deposit two $100 cash amounts in a savings account at two points in time, call them Time zero and Time one, and receive seven percent interest on those deposits, paid and withdrawn at year end (for simplicity), then seven percent would be considered the return on investment. The cash flow for five years, including deposits, interest, and withdrawal of the deposit would be as in Table 1.

Table 1. Cash flow of five year savings account

Time Period (Year)	Cash Flow
0	-100
1	-93 = -100 + (100 x .07)
2	14 = 200 x .07
3	14 = 200 x .07
4	14 = 200 x .07
5	214 = 200 + (200 x .07)

Discounting the cash flow amounts back to time zero at seven percent, it is observed that the discounted cash flow sums to zero, that is, the present value of the cash flow at time zero is equal to zero at the discount rate of seven percent.

The present value of each cash flow at time zero is calculated as in Equation 1.

$$P.V. = \frac{(\text{Cash Flow})_n}{(1 + .07)^n} \qquad (1)$$

n = time period

Therefore, the present value of each cash flow in Time zero would be as in Table 2.

Table 2. Present value of cash flow at seven percent

Time Period (Year)	Cash Flow	P.V. of $(\text{Cash Flow})_t$ in Time zero r=7%
0	-100	-100.00
1	-93	-86.92
2	14	12.23
3	14	11.43
4	14	10.68
5	215	152.58
Total	63	0.00

The discount rate, or AARR, will be converged upon using a computer program, given a cash flow resulting from one combination of price and operating cost schedules. However, instead of using year-end discounting as above, mid-year discounting is used, thus Average Annual Rate of

Return. The present value formula for discounting each cash flow, compounding yearly at mid-year is as in Equation 2.

$$P.V. = \frac{(\text{Cash Flow})_n}{(1+r)^{n-1}(1+r/2)} \tag{2}$$

n = time period

r = discount rate

Therefore, given a set of premises (data) including some combination of price and operating cost schedules, all that is needed to determine AARR is a cash flow. The remainder of Chapter II will be devoted to calculating cash flow from a given data set and determining its associated Average Annual Rate of Return.

C. The Data

The data were provided by a major oil company with investments in the mining industry. The identity of the company will remain anonymous.

Each of the five mining experts had access to various price, cost, and inflation forecasting indexes. The data provided reflects their confidence in those forecasts plus their experience and intuition. Each of the participants had a different assessment of what the price and operating costs would be in the future. Therefore, these five assessments serve as a measure of the uncertainty associated with prices and operating costs. Also, since these opinions are provided by the project evaluators, they are accepted by the evaluation team as a measurement of uncertainty. Furthermore, this type of data can be obtained for most projects from its evaluators. Therefore, similar analysis can be used to compare other projects in terms of profit potential and risk.

Price and Variable Operating Costs, both expressed in dollars per ton of ore mined, were the variables the experts believed presented considerable risk. The five participants treated the remainder of the data as determinate. The determinate data will be assumed certain regardless of the price and operating cost schedules used. Determinate data includes variables such as investment costs, tax rates, and depletion rates.

The first page of data lists all the single value data. This data will remain unchanged over the project life (see Table 3).

The second page of data includes the five experts' assessments of the risk associated with prices. Each expert provided what he considered would be the "most likely" price per ton for uranium in 1981 (the first year of production), the minimum price, and the maximum price. To calculate prices for the remainder of the project life, each expert

Table 3. Single value data

1/01/73	Starting date
23	Number of time periods (excluding 0)
100.00	Working interest, % of cost
5.27	Royalty payment, % of mining revenue
1.089	Production taxes, % of mining revenue
	Working capital requirements:
60	Days accounts receivable (year = 365 days)
40	Days product inventory (year = 365 days)
48.00	Federal income tax rate, % (first periods)
8	Number of periods at above rate
51.00	Federal income tax rate, % (remainder of periods)
22.00	Depletion rate, % of gross revenue
50.00	Depletion rate, % of net revenue
6.50	Depletion overhead rate, % of revenue
15.50	Depletion overhead rate, indirect, % of intangible development costs
1.65	Financial overhead rate, % of working capital
5,836,531	Salvage, $, in Time period 23

supplied percentage escalation rates for years 1982 through 1994, which will be used to escalate their 1981 prices. Escalation rates reflect inflation, changing market conditions, and with respect to costs, increasing project needs (see Table 4).

Also in Table 4 are the average minimum, most likely, and maximum prices for 1981 calculated by averaging the five minimum prices, the five most likely prices, and the five maximum prices. In the same column is the average escalation rate for each year from 1982 to 1994.

In the last column of Table 4 is the most likely price for each year of the project, calculated by Equation 3,

$$(\text{Price})_i = (\text{Price})_{1981} * \left(\prod_{j=1982}^{i} 1 + \left[\left(\begin{matrix} \text{Average} \\ \text{Escalation} \\ \text{Rate, \%} \end{matrix} \right)_j \times .01 \right] \right) \qquad (3)$$

i = year

The third page of data, Table 5, reflects similar treatment of the variable operating cost data.

Table 6 lists investment costs.

Table 7 contains five columns of expenses expressed in gross dollars.

Table 8 includes the production, depreciation, investment tax credit, and outside income after federal income tax

Table 4. Minimum (MIN), most likely (ML), and maximum (MAX) prices ($/ton) for 1981, escalation rates (%) for remainder of project life, averages, and most likely price schedule

Time Period	Expert I	Expert II	Expert III	Expert IV	Expert V	Average Prices, Rates	Most Likely Prices
1981:							
MIN	112.20	100.00	105.00	86.50	96.40	100.02	
ML	186.20	143.64	135.66	122.00	144.07	146.31	146.31
MAX	250.00	205.00	190.00	178.60	220.00	208.72	
1982	7.00	8.00	4.00	2.00	6.00	5.40	154.21
1983	7.00	8.00	4.00	2.00	6.00	5.40	162.54
1984	6.25	8.00	7.00	2.00	6.00	5.85	172.05
1985	6.00	8.00	7.00	2.00	6.00	5.80	182.03
1986	3.00	8.00	7.00	2.00	6.00	5.20	191.49
1987	3.00	8.00	7.00	2.00	6.00	5.20	201.45
1988	0.00	0.00	4.00	1.00	3.00	1.60	204.67
1989	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1990	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1991	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1992	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1993	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1994	0.00	0.00	0.00	0.00	0.00	0.00	204.67
1995							

Table 5. Minimum (MIN), most likely (ML), and maximum (MAX) variable operating costs ($/ton) for 1981, escalation rates (%) for remainder of project life, averages, and most likely cost schedule

Time Period	Expert I	Expert II	Expert III	Expert IV	Expert V	Averages Costs, Rates	Most Likely Prices
1981:							
MIN	53.00	40.00	52.00	32.50	33.00	42.10	
ML	73.29	58.68	71.15	51.25	53.64	61.60	61.60
MAX	92.50	79.50	92.00	73.50	74.00	82.30	
1982	12.00	8.00	9.00	2.50	8.00	7.90	66.47
1983	2.00	8.50	10.00	2.50	8.00	6.20	70.59
1984	0.00	8.50	10.00	2.50	8.00	5.80	74.68
1985	0.00	8.50	9.00	2.50	8.00	5.60	78.86
1986	0.00	8.50	8.00	3.00	8.00	5.50	83.20
1987	1.00	8.50	9.00	2.50	4.65	5.13	87.47
1988	5.30	0.00	5.00	1.00	8.00	3.86	90.85
1989	1.30	0.00	0.00	0.00	4.00	1.06	91.81
1990	0.00	0.00	0.00	0.00	0.00	0.00	91.81
1991	0.00	0.00	0.00	0.00	0.00	0.00	91.81
1992	0.00	0.00	0.00	0.00	0.00	0.00	91.81
1993	0.00	0.00	0.00	0.00	0.00	0.00	91.81
1994	0.00	0.00	0.00	0.00	0.00	0.00	91.81
1995							

Table 6. Investments

Time Period	Depreciable Investment	Intangible Development	Leasehold Investment	Exploration Costs	Working Capital
1973			5,430	10,640	
1974			3,200	98,448	
1975		3,686,000	3,200	207,492	
1976	129,000	5,990,000	3,200	55,000	
1977	12,426,741	5,498,397			10,863
1978	11,215,028	9,437,145			158,657
1979	21,468,235	15,339,617			435,576
1980	22,765,760	4,330,466			4,257,470
1981	14,899,231	4,028,650			-3,575,725
1982	2,437,275	2,593,782			
1983	1,129,513	2,216,866			
1984	1,567,352	1,688,208			
1985	2,023,913	1,794,523			
1986	2,175,706	1,920,382			
1987	1,902,541	178,290			
1988		184,976			
1989					
1990					
1991					
1992					
1993					
1994					
1995					
TOTAL	94,140,295	58,887,302	15,030	371,580	1,286,841

Table 7. Expenses

Time Period	Other Expense	Production Taxes	Financial Overhead	Sales Tax	Ad Valorem Tax
1973	44				
1974	44				
1975	440		60,819		
1976	236,440		161,782	16,427	7,529
1977	1,089,024		457,547	528,386	247,700
1978	1,464,024		798,486	646,407	532,109
1979	2,722,024		1,408,432	1,179,330	1,041,581
1980	4,112,860		1,862,706	946,706	1,443,959
1981	-3,666,765	4,110,750	2,245,265	647,047	1,725,038
1982	476,034	4,110,750	2,475,110	163,006	1,799,747
1983	554,030	6,465,550	2,543,157	108,423	1,849,441
1984	611,189	7,480,550	2,741,587	105,480	1,897,786
1985	579,192	7,480,550	2,886,572	123,717	1,954,490
1986	483,836	7,480,550	2,980,625	132,713	2,015,317
1987	522,332	7,480,550	3,040,840	67,419	2,046,217
1988	543,119	7,480,550	3,071,703	5,993	2,048,964
1989	543,119	7,480,550	3,084,810		2,048,964
1990	543,119	7,480,550	3,084,810		2,048,964
1991	543,119	7,480,550	3,084,810		2,048,964
1992	543,119	7,480,550	3,084,810		2,048,964
1993	543,119	7,480,550	3,084,810		2,048,964
1994	543,119	6,150,900	3,084,810		2,048,964
1995	5,002,640		2,989,971		2,048,964
TOTAL	17,989,221	95,643,450	48,232,448	4,671,054	32,952,626

Table 8. Production, depreciation, investment tax credits, and outside income after taxes

Time Period	Ore Mined (Tons)	Depreciation	Investment Tax Credits	Outside Income After FIT
1973				
1974				
1975				
1976				-190,000
1977				-2,000,000
1978				
1979				
1980				
1981	405,000	11,843,422	7,099,034	
1982	405,000	10,526,469	212,043	
1983	637,000	9,206,819	98,268	
1984	737,000	8,168,562	136,360	
1985	737,000	7,388,615	176,080	
1986	737,000	6,778,764	189,286	
1987	737,000	6,216,988	165,521	
1988	737,000	5,223,208		
1989	737,000	4,395,466		
1990	737,000	3,704,497		
1991	737,000	3,126,496		
1992	737,000	2,642,062		
1993	737,000	2,235,313		
1994	606,000	6,847,121		
1995				
TOTALS	9,423,000	88,303,802	8,076,592	-2,190,000

schedules.

In this chapter, emphasis is placed on demonstrating the derivation of a cash flow and AARR. To do this, the most likely price and most likely operating cost schedules will be used.

D. The Computer Program

Ultimately, 101 cash flows and Average Annual Rates of Return are calculated, the case discussed in this part and 100 more cases in analyzing the uncertainty, which is discussed in Chapter V. To process this huge amount of data, a computer program was written to do the numerous calculations (see Appendix A).

The data in Tables 6, 7, and 8 will remain constant for each of the cases. The calculations which are functions of the changing price and variable operating cost schedules are discussed in the next three sections in the same order as they are calculated by the computer program. A more detailed explanation of the equations in this part is discussed in Appendix B.

E. Determining Income Before Federal Income Taxes

To calculate Income Before Federal Income Taxes, several intermediate calculations must be made, including Gross Revenue, Variable Operating Costs, Total Working Capital, Royalty Expense, Total Financial Overhead, and Total Production Taxes. These are defined by Equations 4-9 (below each independent variable, the table is noted where the data is found).

$$\begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_i = \begin{pmatrix}\text{Ore Mined}\\\text{Table 8}\end{pmatrix}_i * \begin{pmatrix}\text{Most Likely Price}\\\text{Table 4}\end{pmatrix}_i \quad (4)$$

$$\begin{pmatrix}\text{Variable}\\\text{Operating}\\\text{Cost}\end{pmatrix}_i = \begin{pmatrix}\text{Ore Mined}\\\text{Table 8}\end{pmatrix}_i * \begin{pmatrix}\text{Most Likely Var.}\\\text{Op. Cost}\\\text{Table 5}\end{pmatrix}_i \quad (5)$$

$$\begin{pmatrix}\text{Total Working}\\\text{Capital}\end{pmatrix}_i = \begin{pmatrix}\text{Working Capital}\\\text{Table 6}\end{pmatrix}_i$$

$$+ \left(\left[\begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_i - \begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_{i-1} \right] * \begin{pmatrix}\text{Days Accts.}\\\text{Rec. T. 3}\end{pmatrix} /365 \right)$$

$$+ \left(\left[\begin{pmatrix}\text{Total Var.}\\\text{Oper. Costs}\end{pmatrix}_i - \begin{pmatrix}\text{Total Var.}\\\text{Oper. Costs}\end{pmatrix}_{i-1} \right] * \begin{pmatrix}\text{Days Prod.}\\\text{Inven., T. 3}\end{pmatrix} /365 \right) \quad (6)$$

$$\begin{pmatrix}\text{Royalty}\\\text{Expense}\end{pmatrix}_i = \begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_i * \begin{pmatrix}\text{Royalty Payment, \% of}\\\text{Mining Revenue, T. 3}\end{pmatrix} * .01 \quad (7)$$

$$\begin{pmatrix}\text{Total Financial}\\\text{Overhead}\end{pmatrix}_i = \begin{pmatrix}\text{Financial}\\\text{Overhead}\\\text{Table 7}\end{pmatrix}_i + \left[\begin{pmatrix}\text{Total Working}\\\text{Capital}\end{pmatrix}_i * \left(\frac{\text{Fin. Overhead Rate, Table 3}}{100} \right) \right] \quad (8)$$

$$\begin{pmatrix}\text{Total Production}\\ \text{Taxes}\end{pmatrix}_i = \begin{pmatrix}\text{Production Taxes}\\ \text{Table 7}\end{pmatrix}_i + \left[\begin{pmatrix}\text{Gross}\\ \text{Revenue}\end{pmatrix}_i * \begin{pmatrix}\dfrac{\text{Prod. Taxes, \% of Mining Rev., T.3}}{100}\end{pmatrix}\right] \quad (9)$$

i = year

The results of these calculations for each year of the project are listed in Table 9.

"Certain outlays of a corporation are deductible if they are ordinary and necessary expenses paid or incurred during the taxable year in carrying on a trade or business" (1, p. 360). Therefore, to calculate Income Before Federal Income Taxes, Total Expense is subtracted from Gross Revenue, year by year. Total Expense and Income Before Federal Income Taxes are calculated by Equations 10 and 11.

$$\begin{pmatrix}\text{Total}\\ \text{Expense}\end{pmatrix}_i = \begin{pmatrix}\text{Other Expense}\\ \text{Table 7}\end{pmatrix}_i + \begin{pmatrix}\text{Sales Tax}\\ \text{Table 7}\end{pmatrix}_i + \begin{pmatrix}\text{Ad Valorem}\\ \text{Tax, Table 7}\end{pmatrix}_i + \begin{pmatrix}\text{Variable}\\ \text{Operating Costs}\end{pmatrix}_i + \begin{pmatrix}\text{Royalty}\\ \text{Expense}\end{pmatrix}_i + \begin{pmatrix}\text{Total Fin.}\\ \text{Overhead}\end{pmatrix}_i + \begin{pmatrix}\text{Total Production}\\ \text{Taxes}\end{pmatrix}_i \quad (10)$$

Table 9. Calculated schedules of revenue, expenses, and investment

Time Period	Gross Revenue	Variable Oper. Costs	Total Working Capital	Royalty Expense	Total Fin. Overhead	Total Prod. Taxes
1973						
1974						
1975					60,819	
1976					161,782	
1977			10,863		457,726	
1978			158,657		801,104	
1979			435,576		1,415,619	
1980			4,257,470		1,932,955	
1981	59,255,530	24,948,000	8,898,937	3,122,766	2,392,097	4,722,034
1982	62,455,040	26,920,330	742,092	3,291,380	2,487,354	4,755,042
1983	103,537,900	44,965,820	8,730,944	5,456,451	2,687,217	7,533,657
1984	126,800,800	55,039,150	4,927,960	6,682,404	2,822,898	8,788,639
1985	134,156,000	58,119,800	1,546,687	7,070,026	2,912,092	8,864,517
1986	141,128,100	61,318,380	1,496,615	7,437,452	3,005,319	8,936,441
1987	148,468,600	64,465,370	1,551,534	7,824,297	3,066,440	9,012,166
1988	150,841,700	66,956,440	663,099	7,949,361	3,082,644	9,036,648
1989	150,841,700	67,663,960	77,536	7,949,361	3,086,089	9,036,648
1990	150,841,700	67,663,960		7,949,361	3,084,810	9,036,648
1991	150,841,700	67,663,960		7,949,361	3,084,810	9,036,648
1992	150,841,700	67,663,960		7,949,361	3,084,810	9,036,648
1993	150,841,700	67,663,960		7,949,361	3,084,810	9,036,648
1994	124,030,000	55,636,840	-5,725,452	6,536,382	2,990,340	7,430,405
1995			-26,485,660		2,552,057	
TOTAL	1,804,882,170	796,689,930	1,286,604	95,117,324	48,253,792	114,262,789

$$\begin{pmatrix}\text{Income Before Federal}\\\text{Income Taxes}\end{pmatrix}_i = \begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_i - \begin{pmatrix}\text{Total}\\\text{Expense}\end{pmatrix}_i \qquad (11)$$

i = year

With the exception of Total Financial Overhead, all the expenses are "ordinary and necessary" cash transactions. Financial overhead is an "in-house" charge to the project, charged by the corporation, in proportion to the project's investment requirements, including depreciable and intangible investments and working capital. Theoretically, if this project was the only project the corporation was undertaking, then this would be their overhead costs for each year, which would be deductible. So the more capital a project requires, the more overhead it is burdened with in the evaluation stage.

Table 10 shows calculation of Income Before Federal Income Taxes for each year.

F. Determining Taxable Income

The next step in the cash flow calculation is to determine Taxable Income for each year. This is done by determining the allowable deductions for each year and subtracting them from Income Before Federal Taxes.

"Domestic mining companies are permitted an unlimited deduction, on an optional basis, for exploration expenses, provided the amount deducted is recaptured once the mine reaches production stage. Recapture is accomplished by the miner . . . foregoing depletion from the property until deductions foregone equal exploration expenditures previously deducted" (1, p. 366).

Also, "A taxpayer may elect to deduct the cost of developing a mine . . ., whether incurred before or after, the production stage is reached" (1, p. 367).

Thus, exploration and development costs are allowable deductions, provided exploration costs are recaptured from depletion.

The depreciation schedule in Table 8 is the result of depreciating the depreciable investments in Table 6 using the double declining-balance method of depreciation, which is an allowable method (1, p. 404).

Therefore, Tax Deductions are computed by Equation 12.

$$\begin{pmatrix}\text{Tax}\\ \text{Deductions}\end{pmatrix}_i = \begin{pmatrix}\text{Exploration Costs}\\ \text{Table 6}\end{pmatrix}_i + \begin{pmatrix}\text{Intangible Development,}\\ \text{Table 6}\end{pmatrix}_i + \begin{pmatrix}\text{Depreciation}\\ \text{Table 8}\end{pmatrix}_i \qquad (12)$$

i = year

The last allowable deduction is the depletion deduction.

"The basic method of computing depletion is 'cost depletion'. The basis upon which the deduction is allowed is the adjusted basis of the property.

Table 10. Calculation of income before federal income tax and taxable income

Time Period	Gross Revenue	Total Expense	Income Before FIT	Tax Deductions	Depletion Deduction	Taxable Income
1973		44	-44	10,640		-10,684
1974		44	-44	98,448		-98,492
1975		61,259	-61,259	3,893,492		-3,954,751
1976		422,178	-422,178	6,045,000		-6,467,178
1977		2,322,836	-2,322,836	5,498,397		-7,821,233
1978		3,443,643	-3,443,643	9,437,145		-12,880,780
1979		6,358,554	-6,358,554	15,339,610		-21,698,160
1980		8,436,479	-8,436,479	4,330,466		-12,766,940
1981	59,255,530	33,890,190	25,365,344	15,872,070	3,434,568	6,058,704
1982	62,455,040	39,892,880	22,562,160	13,120,250	3,840,786	5,601,123
1983	103,537,900	63,155,020	40,382,944	11,423,680	12,463,760	16,495,480
1984	126,800,800	75,947,520	50,853,328	9,856,770	17,875,020	23,121,520
1985	134,156,000	79,623,800	54,532,288	9,183,138	19,861,230	25,487,900
1986	141,128,100	83,329,420	57,798,704	8,699,146	21,558,650	27,540,890
1987	148,468,600	87,004,220	61,464,416	6,395,278	24,483,000	30,586,120
1988	150,841,700	89,623,150	61,218,624	5,408,184	24,788,190	31,022,240
1989	150,841,700	90,328,120	60,513,648	4,395,466	24,958,110	31,160,060
1990	150,841,700	90,326,840	60,514,928	3,704,497	25,303,600	31,506,810
1991	150,841,700	90,326,840	60,514,928	3,126,496	25,592,600	31,795,320
1992	150,841,700	90,326,840	60,514,928	2,642,928	25,834,810	32,038,040
1993	150,841,700	90,326,840	60,514,928	2,235,313	26,038,190	32,241,400
1994	124,030,000	75,186,030	48,843,984	6,847,121	18,675,040	23,321,800
1995		9,603,661	-9,603,661			-9,603,661
TOTAL	1,804,882,170	1,109,936,408	694,945,762	147,562,669	274,707,554	272,675,539

"Under the percentage depletion method, a flat percentage of gross revenue from the property is taken as the depletion deduction," however, "percentage depletion may not exceed 50 percent of taxable income." Finally, if cost depletion is greater than percentage depletion, then cost depletion is the depletion deduction (1, p. 434).

Therefore, percentage depletion is the lesser of 22 percent of gross revenue (Table 3) and 50 percent of net revenue (Table 3). The depletion deduction is the greater of cost depletion and percentage depletion. Before any deductions are taken, however, exploration costs must be recaptured. The three different depletion calculations are shown in Equations 13-16.

$$\begin{pmatrix}\text{Cost}\\\text{Depletion}\end{pmatrix}_i = \left[\sum_{j=1}^{23}\begin{pmatrix}\text{Leasehold Investments}\\\text{Table 6}\end{pmatrix}_j - \sum_{k=1}^{i-1}\begin{pmatrix}\text{Depletion Deduction}\\\text{before recapture of}\\\text{Expl. Costs}\end{pmatrix}_k\right] * \begin{pmatrix}\text{Ore Mined}\\\text{Table 8}\end{pmatrix}_i \Big/ \sum_{j=i}^{23}\begin{pmatrix}\text{Ore Mined}\\\text{Table 8}\end{pmatrix}_j \qquad (13)$$

$$\begin{pmatrix}\text{Depletion, \%}\\\text{of Gross Rev.}\end{pmatrix}_i = \left[\begin{pmatrix}\text{Gross}\\\text{Revenue}\end{pmatrix}_i - \begin{pmatrix}\text{Royalty}\\\text{Expense}\end{pmatrix}_i\right] * \begin{pmatrix}\text{Depletion Rate,}\\\text{\% Gross Rev.}\\\text{Table 3}\end{pmatrix} * .01 \qquad (14)$$

$$\begin{pmatrix}\text{Depletion, \%}\\ \text{of Net Rev.}\end{pmatrix}_i = \left[\begin{pmatrix}\text{Income Before}\\ \text{FIT, Table 10}\end{pmatrix}_i + \begin{pmatrix}\text{Total}\\ \text{Financial}\\ \text{Overhead}\end{pmatrix}_i - \begin{pmatrix}\text{Depletion}\\ \text{Overhead}\end{pmatrix}_i - \begin{pmatrix}\text{Depreciation}\\ \text{Table 8}\end{pmatrix}_i - \begin{pmatrix}\text{Intangible}\\ \text{Development}\\ \text{Table 6}\end{pmatrix}_i\right] * \begin{pmatrix}\text{Depletion Rate, \%}\\ \text{of Net Revenue}\\ \text{Table 3}\end{pmatrix} * .01 \quad (15)$$

i = year

$$\begin{pmatrix}\text{Depletion}\\ \text{Overhead}\end{pmatrix}_i = \left[\begin{pmatrix}\text{Gross}\\ \text{Revenue}\end{pmatrix}_i - \begin{pmatrix}\text{Royalty}\\ \text{Expense}\end{pmatrix}_i\right] * \begin{pmatrix}\text{Depl. overhead rate}\\ \text{\% of revenue}\\ \text{Table 3}\end{pmatrix} * .01 + \left[\begin{pmatrix}\text{Intangible}\\ \text{Development}\\ \text{Table 6}\end{pmatrix}_i * \begin{pmatrix}\text{Depl. overhd.}\\ \text{rate, indirect}\\ \text{Table 3}\end{pmatrix} * .01\right] \quad (16)$$

i = year

In calculating percentage depletion, Gross Revenue was reduced by Royalty Expense for each calculation because the lessor is allowed a depletion deduction on the royalty paid to him. In calculating Depletion, % of Net Revenue, Financial Overhead is added back to Income Before Federal Income Taxes and then reduced by Depletion Overhead since, in practice, overhead for depletion purposes is measured as a function of revenue from the mine and development costs, not according to the company's procedure.

Taxable Income can now be calculated by Equation 17.

$$\begin{pmatrix}\text{Taxable}\\ \text{Income}\end{pmatrix}_i = \begin{pmatrix}\text{Income Before}\\ \text{FIT, Table 10}\end{pmatrix}_i - \begin{pmatrix}\text{Tax}\\ \text{Deductions}\end{pmatrix}_i - \begin{pmatrix}\text{Depletion}\\ \text{Deduction}\end{pmatrix}_i \qquad (17)$$

i = year

Taxable Income, Tax Deductions, and Depletion Deduction results are shown in Table 10.

G. Determining Cash Flow and Average Annual Rate of Return

Finally, the cash flow for each year of the project is calculated using Equations 18-22.

$$\begin{pmatrix}\text{Net}\\ \text{Tax}\end{pmatrix}_i = \left[\begin{pmatrix}\text{Taxable}\\ \text{Income}\end{pmatrix} * \begin{pmatrix}\text{\% Tax Rate,}\\ \text{Table 3}\end{pmatrix} * .01\right] - \begin{pmatrix}\text{Investment Tax}\\ \text{Credit, Table 8}\end{pmatrix}_i \qquad (18)$$

$$\begin{pmatrix}\text{Income After}\\ \text{Federal Income Taxes}\end{pmatrix}_i = \begin{pmatrix}\text{Income Before}\\ \text{FIT}\end{pmatrix}_i - \begin{pmatrix}\text{Net}\\ \text{Tax}\end{pmatrix}_i \qquad (19)$$

$$\begin{pmatrix}\text{Cash}\\ \text{Income}\end{pmatrix}_i = \begin{pmatrix}\text{Income After}\\ \text{FIT}\end{pmatrix}_i + \begin{pmatrix}\text{Salvage,}\\ \text{Table 3}\end{pmatrix}_i$$

$$+ \begin{pmatrix}\text{Outside Income}\\ \text{After FIT, T. 8}\end{pmatrix}_i + \begin{pmatrix}\text{Return of}\\ \text{Working Capital}\end{pmatrix}_i \tag{20}$$

$$\begin{pmatrix}\text{Total}\\ \text{Investment}\end{pmatrix}_i = \begin{pmatrix}\text{Depreciable}\\ \text{Investment}\\ \text{Table 6}\end{pmatrix}_i \begin{pmatrix}\text{Intangible}\\ \text{Development}\\ \text{Table 6}\end{pmatrix}_i$$

$$+ \begin{pmatrix}\text{Leasehold}\\ \text{Investment}\\ \text{Table 6}\end{pmatrix}_i + \begin{pmatrix}\text{Exploration}\\ \text{Costs, Table 6}\end{pmatrix}_i$$

$$+ \begin{pmatrix}\text{Total Working}\\ \text{Capital}\\ \text{Table 9}\end{pmatrix}_i \tag{21}$$

$$\begin{pmatrix}\text{Cash}\\ \text{Flow}\end{pmatrix}_i = \begin{pmatrix}\text{Cash}\\ \text{Income}\end{pmatrix}_i - \begin{pmatrix}\text{Total}\\ \text{Investment}\end{pmatrix}_i \tag{22}$$

i = year

In calculating Cash Income, Return of Working Capital is the sum of the Total Working Capital for the project, returned in the last year of the project, when all outstanding debts or accounts receivable are settled. Therefore, this is the only year there will be Return of Working Capital.

The calculated values for Net Tax, Income After Federal Income Taxes, Cash Income, Total Investment and Cash Flow are listed in Table 11.

Note that negative taxes are calculated, not carried

Table 11. Calculation of cash flow

Time Period	Net Tax	Income After FIT	Salvage, Outside Income, Return of Working Capital	Cash Income	Total Investment	Cash Flow
1973	-5,128	5,084		5,084	16,070	-10,986
1974	-47,276	47,232		47,232	101,648	-54,416
1975	-1,898,280	1,837,021		1,837,021	3,896,692	-2,059,671
1976	-3,104,245	2,682,067	-190,000	2,492,067	6,177,200	-3,685,133
1977	-3,754,191	1,431,355	-2,000,000	-568,645	17,935,980	-18,504,620
1978	-6,182,778	2,739,135		2,739,135	20,810,810	-18,071,680
1979	-10,415,117	4,056,563		4,056,563	37,243,420	-33,186,840
1980	-6,128,133	-2,308,346		-2,308,346	31,353,680	-33,662,010
1981	-4,009,096	29,374,440		29,374,430	27,826,800	1,547,632
1982	2,644,529	19,917,631		19,917,610	5,773,149	14,144,460
1983	8,314,426	32,068,518		32,068,510	12,077,320	19,991,180
1984	11,655,614	39,197,714		39,197,710	8,183,520	31,014,190
1985	12,822,750	41,709,538		41,709,530	5,365,123	36,344,400
1986	13,856,570	43,942,134		43,942,120	5,592,703	38,349,420
1987	15,433,403	46,031,013		46,031,000	3,632,365	42,398,640
1988	15,821,342	45,397,282		45,397,280	848,075	44,549,200
1989	15,891,632	44,622,016		44,622,010	77,536	44,544,480
1990	16,068,475	44,446,453		44,446,440		44,446,440
1991	16,215,869	44,299,059		44,299,050		44,299,050
1992	16,339,404	44,175,524		44,175,520		44,175,520
1993	16,443,117	44,071,811		44,071,800		44,071,800
1994	11,894,121	36,949,863		36,949,850	-5,725,452	42,675,290
1995	4,897,867	-4,705,794	7,123,135	2,417,341	-26,485,660	28,902,990
TOTAL	122,807,667	572,138,095	4,933,135	577,071,230	154,700,979	422,370,251

forward or backward. This is because of the assumption of consolidation. This project will contribute "tax credits" to the overall operations of the company. Therefore, to accurately measure the full contribution of the project, taxes are allowed to become negative.

Converging on a discount rate which results in the discounted cash flow summing to zero, it is found that that discount rate is .17802. Therefore, the Average Annual Rate of Return equals 17.802 percent, given this set of premises.

H. Interpretation of Average Annual Rate of Return

Considering that the "most likely" price and variable operating cost schedules (Tables 4 and 5) were used in determining this AARR of 17.802 percent, if there was to be only one measure of the profit potential of the project, then 17.802 percent would be a logical choice, since it would reflect the five expert's estimation of the "most likely" AARR.

Average Annual Rate of Return should not be confused with Return on Assets (ROA). AARR is a measure of return over the project life. ROA is a yearly measurement of return on capital employed, calculated by dividing financial net income for each year by the average capital employed in that year.

In Chapter V, the uncertainty expressed by the five participating mining experts will be dealt with in length, using the discounted cash flow method described here to generate one hundred possible (probable) AARR's.

III. ECONOMIC EVALUATION AT PRESENT

Currently, most companies conduct economic feasibility studies by employing computer programs that use the discounted cash flow method described in the previous part. These programs supply the project evaluators with single point estimates of return on investment (AARR). The programs are much more complex and flexible than the one used for this particular data case but ultimately, a cash flow is calculated and discounted to determine the AARR.

Uncertainty is seldom considered and when it is, it is dealt with only to the extent of making several runs using different price and operating cost schedules, which provides the project evaluators with a sensitivity-type study of the proposed project; the sensitivity of AARR to differing prices and other variables. Typically, the project recommended for funding is chosen by these few points and intuition. The project selected by the Executive Committee (or some other allocative committee for investment) is usually chosen on the basis of the past performance of the manager recommending the project, in particular, the reliability of his estimates in the past.

For the manager, selecting the project(s) to recommend for funding is very difficult. If perfect knowledge existed, then the actual AARR the project would yield would be known and the decision would be trivial. In the absence of perfect knowledge, projects could be compared fairly easily if a state of "risk" existed which is when all the outcomes (AARR's) and probabilities associated with each outcome are known (2). Positive real numbers might be considered the possible outcomes or AARR's. Clearly, however, the probability of each of the outcomes (or outcome intervals) is not known and the manager is, therefore, dealing with "uncertainty".

What the manager needs is a method, suitable for computer programming, that provides a rapid, relatively simply to use, and intuitively appealing means of representing numerically and graphically the risk associated with each project. Specifically, a probability distribution function on AARR would be most useful.

A good deal of work has been done to provide managers with probability distribution functions on AARR but, as will be shown in reviewing the literature, the results have been unsatisfactory.

IV. REVIEW OF THE LITERATURE

A review of the literature reveals that several problems confront economic analysts when dealing with uncertainty. First, the success of using econometric models for forecasting future values of such variables as inflation rates will be discussed. Next, two articles discussing the use and success of risk analysis tools are summarized. Lastly, three reports on the "Delphi" method are summarized, which is of particular interest to this paper since the Delphi method involves the use of a panel of experts to extract information (forecasts).

Economic analysts often rely on econometric models to obtain forecasts for variables of interest in their studies, however, H. A. Merklein concludes in a recent article that "econometric forecasting is not good theory and it is not good practice" (3). Merklein refers to an article by S. K. McKnees where the five most widely known and influential econometric models were evaluated (4). These five include the Wharton model, the U.S. Department of Commerce model, the Chase Econometrics model, the Data Resources model, and the National Bureau of Economic Research model.

Merklein listed several examples of forecasting errors, including errors in forecasted inflation rates and unemployment rates. For instance, the five models agreed in the first quarter of 1970 that the inflation rate would fall within the narrow range of 3.0 to 4.0%. The actual inflation rate turned out to be 5.0%. Similarly, the low and high forecasted unemployment rates were 4.2% and 5.2%, respectively. The actual was 6.0%.

Merklein continued to point out several other errors, one of particular interest to energy economists. The one-year inflation forecast immediately preceding the oil embargo was 4.4 to 5.4%. After revision for increased crude-oil prices, the forecasts were 5.6 to 8.3%. The actual inflation in the third quarter of 1974 was 10.5%.

Compounding the problem that forecasting tools are unsatisfactory, risk analysis tools and methodologies have also proven unsuccessful. William K. Hall writes "that despite its popularity among researchers and managers alike, Risk Analysis as a manangement tool has not been successful" (5, p. 25). His basic conclusion is that risk analysis is not having - and will not have - a measurable, positive impact on the planning process in large firms.

Hall's conclusion is drawn from empirical observations from two sources, including in-depth studies of the decision-making process leading to major new product investments in four "Fortune 500" manufacturing firms and interviews with senior managers in twelve large firms operating as either manufacturing, utility, or financial institutions.

Hall also referred to a paper by E. S. Carter (6). Carter suggests that the degree of success with risk analysis will depend on the resolution of the following questions:

1. How mature was the risk analysis technique when each company decided to use it?
2. What were the origins of the decision to adopt it?
3. How was it fitted in with company organization?
4. How were managers who were supposed to use and benefit from the technique prepared for handling it?
5. How were the data generated, and how were they put together in model form?

Management must resolve the first four questions. Chapter V of this thesis provides a resolution to question five.

Two more problems the analyst faces according to Hall are: (1) the decision as to who should quantify uncertainty and how this should be done and, (2) the decision as to what uncertainties should be quantified. These questions are also answered by the procedure in Chapter V.

One technique often used is Monte Carlo simulation, however, L. B. Davidson and D. O. Cooper point out that it has not turned out as well as expected (7). Its drawbacks are: (1) it can be expensive to use; (2) the time and effort required to get an answer can be excessive; and (3) it can be too "black-boxish".

These two authors propose a "Parameter-Method Procedure". The result is a probability distribution of present value

($ million) rather than a percentage return on investment.

This thesis approaches uncertainty through the use of a panel of experts, e.g., the project evaluators. The Rand Corporation has conducted experiments using panels of experts for forecasting and have labeled it the "Delphi Method".

"Delphi is the name of a set of procedures for eliciting and refining the opinions of a group of people" (8). The procedure has three distinctive characteristics:

1. Anonymity;
2. Controlled feedback; and
3. Statistical "group response".

The Delphi process has had several applications. One potential application is as a business forecasting tool (9).

Bowerman and O'Connell have summarized the Delphi Method and its use (10). It assumes that the panel members are recognized "experts" in the field of interest, and it also assumes that the combined knowledge of the panel members will produce predictions at least as good as those that would be produced by one member. The panel members are physically separated. Each participant is asked to respond to a series of questionnaires and to return the completed questionnaire to a panel coordinator. After the first questionnaire is completed, subsequent questionnaires are accompanied by information concerning the opinions of the group as a whole.

Thus, the participants can review their predictions relative to the group response. It is hoped that after several rounds of questionnaires the group response will converge on a concensus that can be used.

The method used in this paper for extracting information from a panel of experts is similar to the Delphi Method in some respects, but there are also several differences, for instance, there will be only one round of questionnaires.

In conclusion, quantitative tools such as econometric models and risk analysis procedures have proven to be unsatisfactory. Qualitative tools such as the Delphi Method offer possible solutions for business forecasting. But, regardless of whether managers can obtain risk profiles for each project, the decision as to which project to recommend must still be made. These decisions are made on the basis of project evaluators' expertise. As will be seen in the development of the statistical procedure, the focus of this paper is on that decision and their expertise.

V. THE STATISTICAL PROCEDURE

A. Introduction

Using a panel of experts to extract forecasts of future prices for uranium ore and variable operating costs for the mine, a frequency distribution, similar to a probability distribution function, will be formed on AARR. The frequency distribution will not be the actual probability distribution function on AARR; it will be the underlying collective opinion as to what the panel of experts believe to be the probability distribution function.

Similar treatment of other proposed projects will facilitate the comparison of projects on a risk basis. Although the comparisons or "ordering" of the projects may be inaccurate in actuality, they will be consistent with the estimates given by the experts. In terms of usefulness, estimating risk as a function of expert forecasts is much more acceptable to those same experts than an abstract model that is neither understood nor trusted. As was pointed out in reviewing the literature, their mistrust is often warranted. ("Consistency" will be discussed in Section V.E.)

The crucial part in the statistical method is the gathering of the data, which is discussed first.

Next, the calculations will be described in detail, concentrating only on the mathematics and not the justification for each step. In the next section, a "package" of results will be presented with interpretations.

In the last section, a critical evaluation of the procedure is made, pointing out both its virtues and shortcomings and discussing the assumptions. Lastly, further work to be done in economic feasibility evaluation and several extensions to the procedure are suggested.

B. Procedure for Gathering the Data

The emphasis of this paper is placed on gathering the data. The procedure for collecting the data is relatively simple and should be appealing to the experts.

First, a committee and chairperson are established to make two preliminary decisions. This committee might typically be composed of the project evaluators. If a committee is not used, a "panel coordinator" will need to be designated. It will be his/her responsibility to coordinate the uncertainty analysis. The committee or panel coordinator must determine:

1. The data that will be treated as determinate and the independent variables that contribute significant uncertainty to the evaluation,

2. Who the recognized experts are for forecasting the uncertain variables. Experts will typically be field experts and project evaluators. (It is assumed a panel coordinator will not be biased in selecting the experts, for instance, selecting only those experts whose estimates coincide with his/her own.)

Ultimately, it is the responsibility of the committee or coordinator to select variables that are independent. The question one might ask if two variables are independent is: Considering the range over which each variable may take on value, given a value of one, is the other variable free to take on any value in its range? If yes, they are independent. If one variable is restricted by the value of another, they are not independent. Correlation coefficients among the variables are provided in the statistical procedure to confirm or cast suspicion on whether the variables are independent (see Figure 5).

If two variables are not independent, then only one of the two should be evaluated as an uncertain variable. The other should be incorporated in the determinate data as a function of the variable(s) of which it is dependent. For instance, royalty payments and production taxes (Table 3) are functions of mining revenue and, therefore, price. Royalty payments and productions taxes might be considered uncertain, however, they are uncertain only insofar as price is uncertain. Therefore, royalty payment and production tax rates are considered as determinate, independent data.

After determining the uncertain variables and the experts to question, the next step is to fill out the questionnaires (data forms).

Prior to filling out the data forms, discussion among the experts is allowed and encouraged. Discussion topics might include literature, forecasts made by econometric models and by other experts. Also, if one expert is "more expert" than others, then other experts might seek his opinion. This is simply more information and is desirable. Therefore, experts of higher recognized caliber will influence the uncertainty analysis more so than others.

The experts will then be physically separated when making their forecasts. The experts will remain anonymous, free to make whatever forecasts they believe are most reasonable. The experts are separated to avoid dominating personalities that might influence and distort all the forecasts. The separation should be appealing to the experts and encourage their best efforts in making their forecasts.

An implicit assumption is that no expert(s) will sabotage the analysis by providing unrealistic forecasts, for instance, to make the project seem more profitable than it actually will be.

Each expert need not forecast all the uncertain variables, only those which he/she is considered an expert to estimate.

If the variable is single value data, each expert will provide his/her forecasts, including the minimum, most likely, and maximum values the variable might take on.

Typically, the uncertain variables will be in the form of schedules with changing values over time. In this case, each expert will forecast the minimum, most likely, and maximum values the variable might take on in the first relevant time period. The experts will then provide escalation rates for the remainder of the project life for each variable. The escalation rates need not be equivalent for all variables.

With respect to this problem, recalling Tables 4 and 5, the uncertain variables are price ($1/ton) and variable operating costs ($/ton). The minimum, most likely, and maximum values in the first year of production were provided and escalation rates thereafter. The determinate data are the data in Tables 3, 6, 7, and 8. The data were collected in similar fashion as discussed above.

C. The Mathematics

In this section only the mathematical calculations will be presented. The assumptions of the method will be presented first. The statistical method and assumptions will be scrutinized in the Critical Evaluation, Section V.E.

The assumptions are:

1. The experts have not biased the data by including only experts that agree with one another or padding forecasts to improve AARR's.

2. The uncertain variables are statistically independent of one another and the determinate data.

3. There is no uncertainty with respect to the determinate data. The probability of occurrence of each piece of determinate data is equal to one, regardless of the values the uncertain variables take on.

4. There is no uncertainty with respect to relative rates of future escalation (inflation). The escalation rate for each variable at each time period will be the average of the rates provided by the experts.

Before beginning the calculations that ultimately result in the distribution on AARR, preliminary calculations will be made in relation to the assumption of independence among the uncertain variables. These calculations will be linear correlation coefficients among the uncertain variables using the "most likely" forecasts.

The purpose of the correlation coefficients is to confirm the independence of the uncertain variables or to cast doubt on their independence. In this problem, price and variable operating costs are the two variables under investigation. Each of the five experts provided an estimate of the most likely price and variable operating cost for 1981 (see Table 12). If an expert had not provided a forecast for both variables, then his single forecast would not be used in testing for independence.

The Pearson correlation coefficient is given by Equation 23.

$$r = \left[\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})\right] / \left[\sum_{i=1}^{n}(X_i-\overline{X})^2 \sum_{i=1}^{n}(Y_i-\overline{Y})^2\right]^{1/2} \qquad (23)$$

Table 12. Most likely prices (P) and variable operating costs (VC) by each expert

	Expert I	Expert II	Expert III	Expert IV	Expert V
P	186.20	143.64	135.66	122.00	144.07
VC	73.29	58.68	71.15	51.25	53.64

For this problem,

$$r_1 = [\sum_{i=1}^{5} (P_i-\overline{P})(VC_i-\overline{VC})]/[\sum_{i=1}^{5} (P_i-\overline{P})^2 \sum_{i=1}^{5} (VC_i-\overline{VC})^2]^{1/2}$$

$$= .662$$

r_1 estimates ρ, the actual degree of correlation between P and VC. For P and VC to be perfectly independent (uncorrelated) ρ would equal zero. Therefore, a test statistic, t_{c_1}, is computed to test the null hypothesis that $\rho=0$ against the alternative hypothesis that $\rho\neq0$. t_c is calculated by Equation 24.

$$t_{c_1} = r_1/[(1-r_1^{\ 2})/(n-2)]^{1/2} = 1.530 \qquad (24)$$

n = number of observations = 5.

$t_{c_1} \sim t_{(n-2)}$ degrees of freedom

From a student's t table, the critical value for the two-tailed test with a 95% confidence level (.025 area in each tail) and 3 degrees of freedom is 3.182. Since $|t_c| < 3.182$, the test fails to reject the hypothesis that $\rho=0$. If $|t_c| > 3.182$, then the hypothesis that $\rho=0$ would be rejected and $\rho\neq0$ would be accepted, which would cast serious doubt on the assumption that P and VC are independent.

Failing to reject the null hypothesis that $\rho=0$ will be a sufficient condition for independence, but not a necessary one. (Recall that ultimately the responsibility for choosing independent uncertain variables rests with the experts.)

Turning now to forming a frequency distribution on AARR, the technique will be to determine one hundred possible AARR's, each having probability of occurrence equal to one one-hundredth, and from these, form a frequency distribution to approximate a continuous probability density function.

The one hundred AARR's will be generated simply and intuitively. First, ten price schedules and ten variable operating cost (VC) schedules will be determined, each of which will be assigned a probability of occurrence of one-tenth. There will be 10 x 10 = 100 combinations of price and VC schedules. For each price and VC schedule combination, the cash flow computer program described in Chapter II will be used to determine that combination's AARR. Since prices, VC, and the determinate data are all assumed to be statistically independent, the probability of that AARR's occurrence will be one one-hundredth.

Therefore, the calculational procedure will begin by forming triangular probability distribution functions with the price and VC data for 1981. Triangular distributions are used to allow for nonsymmetric distributions and computational ease. To form the triangular distributions, minimum (MIN), most likely (ML), and maximum (MAX) values are needed for both prices and VC. The logical choice for these are the average minimum, most likely and maximum values for prices and VC shown in Tables 4 and 5, respectively. The height (H) of the triangular distribution is given by Equation 25. Thus, the triangular distribution has area equal to one. The

$$H = 2/(MAX-MIN) \tag{25}$$

triangular distributions would be similar to those in Figures 1 and 2.

Since the ten price schedules and ten VC schedules will be calculated in similar fashion from here, for illustrative purposes, the analysis will now focus only on the price triangular distribution.

The triangular distribution will be transformed into a histogram-type distribution with each column containing area (probability) equal to one-tenth. Several techniques can be used to accomplish this. The procedure which will be shown

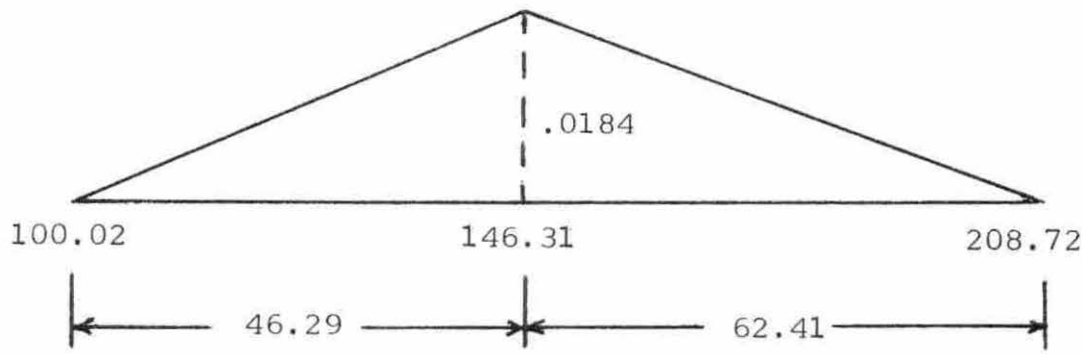

Figure 1. Price triangular probability distribution function

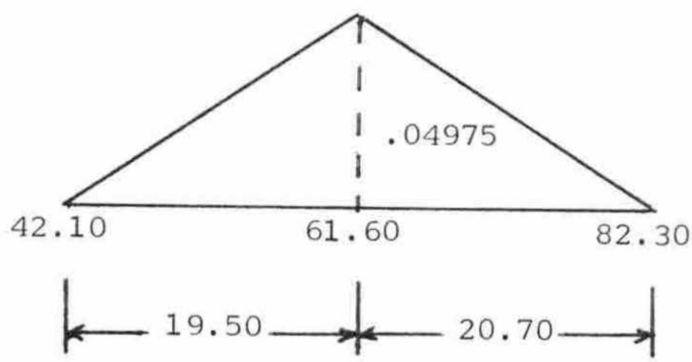

Figure 2. VC triangular probability distribution function

here was chosen since it would be relatively easy to program.

The price triangle is first divided into two right triangles as in Figure 3a. The slope of the hypotenuse of Triangle I, m_1, is .0184 ÷ 46.29 = .0003975, and for Triangle II, m_2 = .0002948.

To yield an area equal to one-tenth in each segment, the integration reduces from Equation 26 to Equation 27.

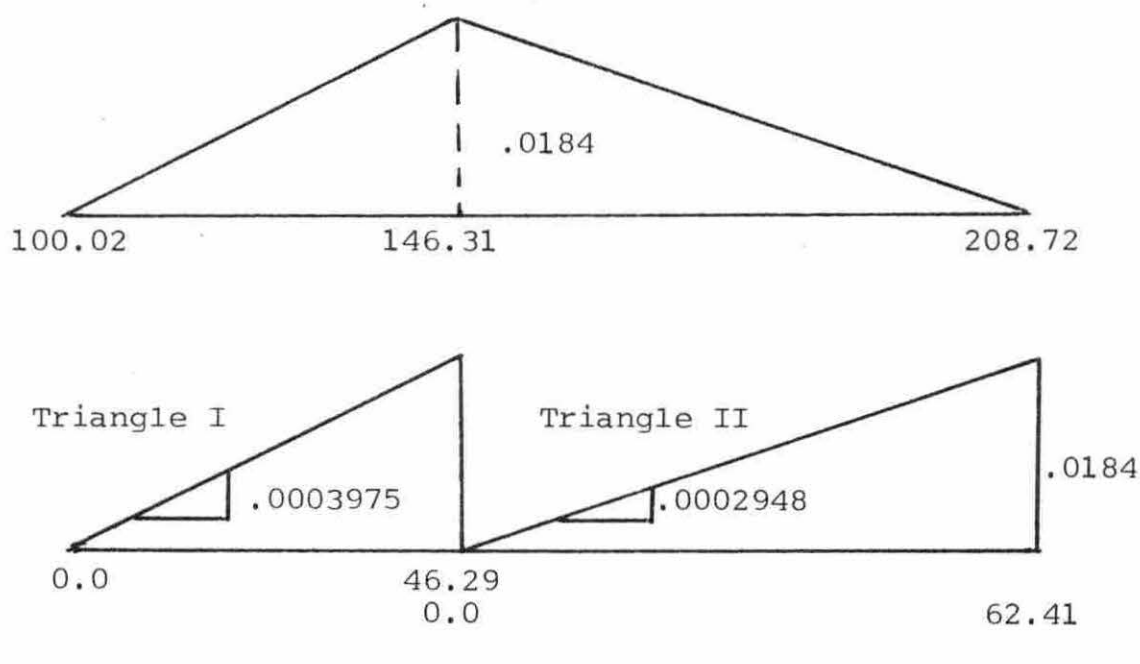

Figure 3a. Division of price triangle

$$\int_L^U f(x)\ dx = \int_L^U m\ x\ dx = m \int_L^U x\ dx = .01 \qquad (26)$$

$$x^2/2]_L^U = (U^2-L^2)/2 = (1/m) \times .01$$

$$U^2 = (2/m) \times .01 + L^2$$

$$U = [(.2/m) + L^2]^{1/2} \qquad (27)$$

For Triangle I, from Equation 27, $U = [(.2/.0003975) + L^2]^{1/2} = (503.14 + L^2)^{1/2}$. Both triangles are integrated consecutively from left to right, the upper limit (U) becoming the lower limit (L) in each consecutive calculation. Integration continues so long as the upper limit (U) is less than or equal to the length of the base of the triangle.

The upper limits for Triangle I are found to be

$U_1 = 0$,

$U_2 = (503.14 + 0.00^2)^{1/2} = 22.43$,

$U_3 = (503.14 + 22.43^2)^{1/2} = 31.72$,

$U_4 = (503.14 + 31.72^2)^{1/2} = 38.85$,

$U_5 = (503.14 + 38.85^2)^{1/2} = 44.86$.

The upper limits of Triangle II are

$U_1 = 0$,

$U_2 = [(.2/.0002948) + L^2]^{1/2} = (678.43 + 0.00^2)^{1/2} = 26.05$,

$U_3 = (678.43 + 26.05^2)^{1/2} = 36.84$,

$U_4 = (678.43 + 36.84^2)^{1/2} = 45.11$,

$$U_5 = (678.43 + 45.11^2)^{1/2} = 52.09,$$
$$U_6 = (678.43 + 52.09^2)^{1/2} = 58.24.$$

Returning to the original price triangular distribution, the upper limits of Triangle I are added to 100.02 to find the interval boundaries to the left of the most likely price, and the upper limits of Triangle II are subtracted from 208.72 to find the interval boundaries to the right of the most likely price. The interval boundaries are shown in Figure 3b.

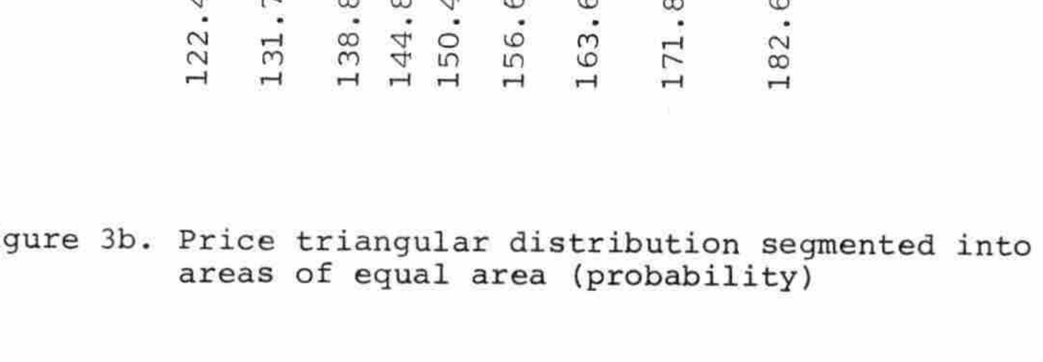

Figure 3b. Price triangular distribution segmented into ten areas of equal area (probability)

The ten price schedules, each assigned probability equal to one-tenth, are determined by using the midpoint of each of intervals in Figure 4 for the 1981 price and, thereafter, prices are calculated using Equation 3 in Section II.C and the average escalation rates listed in Table 4. The ten price schedules are in Tables 13 and 14.

The variable operating cost data were handled in similar fashion as the price data. The resulting ten variable operating cost schedules are listed in Tables 15 and 16.

The cash flow program described in Chapter II was then used to calculate one hundred AARR's using all one hundred combinations of price and variable operating cost schedules. The matrix of AARR's is shown in Figure 4.

The expected, or mean, AARR is 18.18 percent and the variance of the AARR's is 39.83 (see Equations 28 and 29).

$$\text{Mean AARR} = \overline{\text{AARR}} = \left(\sum_{VC=1}^{10} \sum_{P=1}^{10} \text{AARR}_{P,VC} \right) / 100 \tag{28}$$

$$\text{Var(AARR)} = \left[\sum_{VC=1}^{10} \sum_{P=1}^{10} (\text{AARR}_{P,VC} - \overline{\text{AARR}})^2 \right] / 100 \tag{29}$$

Holding price constant at each of the ten price schedules and varying VC over all ten schedules at each of these ten price schedules, the mean or expected AARR, given a price schedule, can be calculated using Equation 30. These values can be found in the extreme right column of Figure 4.

Table 13. Price schedules one through five

Time Period	P_1	P_2	P_3	P_4	P_5
1973					
1974					
1975					
1976					
1977					
1978					
1979					
1980					
1981	111.23	127.08	135.29	141.86	147.67
1982	117.24	133.94	142.60	149.52	155.64
1983	123.57	141.18	150.30	157.59	164.05
1984	130.80	147.43	159.09	166.81	173.65
1985	138.38	158.10	168.32	176.39	183.72
1986	145.58	166.32	177.07	185.67	193.27
1987	153.14	174.97	186.28	195.32	203.32
1988	155.60	177.77	189.26	198.45	206.57
1989	155.60	177.77	189.26	198.45	206.57
1990	155.60	177.77	189.26	198.45	206.57
1991	155.60	177.77	189.26	198.45	206.57
1992	155.60	177.77	189.26	198.45	206.57
1993	155.60	177.77	189.26	198.45	206.57
1994	155.60	177.77	189.26	198.45	206.57
1995					

Table 14. Price schedules six through ten

Time Period	P_6	P_7	P_8	P_9	P_{10}
1973					
1974					
1975					
1976					
1977					
1978					
1979					
1980					
1981	153.56	160.12	167.75	177.28	195.70
1982	161.85	168.77	176.81	186.85	206.27
1983	170.59	177.88	186.36	196.94	217.41
1984	180.57	188.29	197.26	208.46	230.12
1985	191.05	199.21	208.70	220.56	243.47
1986	200.98	209.57	219.55	232.02	256.13
1987	211.43	220.46	230.97	244.09	269.45
1988	214.81	223.99	234.66	247.99	273.76
1989	214.81	223.99	234.66	247.99	273.76
1990	214.81	223.99	234.66	247.99	273.76
1991	214.81	223.99	234.66	247.99	273.76
1992	214.81	223.99	234.66	247.99	273.76
1993	214.81	223.99	234.66	247.99	273.76
1994	214.81	223.99	234.66	247.99	273.76
1995					

Table 15. Variable operating cost schedules one through five

Time Period	VC_1	VC_2	VC_3	VC_4	VC_5
1973					
1974					
1975					
1976					
1977					
1978					
1979					
1980					
1981	45.23	52.78	56.03	58.63	60.86
1982	48.80	56.95	60.46	63.26	65.67
1983	51.83	60.48	64.20	67.18	69.74
1984	54.84	63.99	67.93	71.08	73.78
1985	57.91	67.57	71.73	75.06	77.92
1986	61.09	71.29	75.68	79.19	82.20
1987	64.22	74.95	79.56	83.25	86.42
1988	66.70	77.84	82.63	86.47	89.75
1989	67.41	78.66	83.51	87.38	90.71
1990	67.41	78.66	83.51	87.38	90.71
1991	67.41	78.66	83.51	87.38	90.71
1992	67.41	78.66	83.51	87.38	90.71
1993	67.41	78.66	83.51	87.38	90.71
1994	67.41	78.66	83.51	87.38	90.71
1995					

Table 16. Variable operating cost schedules six through ten

Time Period	VC_6	VC_7	VC_8	VC_9	VC_{10}
1973					
1974					
1975					
1976					
1977					
1978					
1979					
1980					
1981	62.98	65.28	67.95	71.29	77.74
1982	67.96	70.44	73.32	76.92	83.88
1983	72.17	74.80	77.86	81.69	89.08
1984	76.35	79.14	82.38	86.43	94.25
1985	80.63	83.57	86.99	91.27	99.53
1986	85.06	88.17	91.78	96.29	105.00
1987	89.43	92.69	96.49	101.23	110.39
1988	92.88	96.27	100.21	105.14	114.65
1989	93.87	97.29	101.27	106.25	115.86
1990	93.87	97.29	101.27	106.25	115.86
1991	93.87	97.29	101.27	106.25	115.86
1992	93.87	97.29	101.27	106.25	115.86
1993	93.87	97.29	101.27	106.25	115.86
1994	93.87	97.29	101.27	106.25	115.86
1995					

VC / PRICE	VC_1	VC_2	VC_3	VC_4	VC_5	VC_6	VC_7	VC_8	VC_9	VC_{10}	AVERAGES
P_1	13.59	10.77	9.47	8.36	7.36	6.37	5.23	3.84	1.97	0.10	6.71
P_2	17.92	15.55	14.47	13.56	12.76	11.97	11.12	10.08	8.72	5.80	12.20
P_3	19.86	17.70	16.71	15.88	15.16	14.44	13.65	12.69	11.46	8.94	14.65
P_4	21.24	19.28	18.35	17.58	16.91	16.24	15.51	14.63	13.48	11.12	16.43
P_5	22.39	20.60	19.72	18.99	18.35	17.73	17.04	16.21	15.13	12.92	17.91
P_6	23.47	21.85	21.03	20.34	19.73	19.14	18.49	17.71	16.70	14.63	19.31
P_7	24.60	23.13	22.40	21.76	21.18	20.63	20.01	19.27	18.32	16.39	20.77
P_8	25.84	24.50	23.85	23.29	22.77	22.25	21.67	20.98	20.09	18.29	22.35
P_9	27.30	26.07	25.50	25.01	24.56	24.12	23.60	22.95	22.13	20.47	24.17
P_{10}	29.93	28.81	28.32	27.92	27.55	27.20	26.81	26.32	25.66	24.22	27.27
AVERAGES	22.61	20.83	19.98	19.27	18.63	18.01	17.31	16.47	15.37	13.29	18.18

Figure 4. Matrix of one hundred AARR's (%)

$$\text{Mean } AARR_{\overline{P}_i, VC} = E(AARR_{\overline{P}_i}) = (\sum_{VC=1}^{10} AARR_{\overline{P}_i, VC})/10 \qquad (30)$$

$i = 1,2,\ldots,10$

$\overline{P}_i$ indicates the price schedule is fixed at i

Likewise, holding variable operating costs constant at each of the variable operating cost schedules and varying prices, the expected AARR, given a VC schedule, can be calculated by Equation 31. These values can be found on the bottom

$$\text{Mean } AARR_{P,\overline{VC}_i} = E(AARR_{\overline{VC}_i}) = (\sum_{P=1}^{10} AARR_{P,\overline{VC}_i})/10 \qquad (31)$$

$i = 1,2,\ldots,10$

$\overline{VC}_i$ indicates the VC schedule is fixed at i

row of Figure 4.

The results of these last two sets of calculations will be graphed and presented in the Results and Interpretation section (Section V.D).

This concludes the necessary calculations. The results are summarized in the next section. Included will be a frequency table, a graph of the frequency distribution, and several other results of interest.

D. Results and Interpretation

In this section, a "package" of results will be presented for the benefit of the user or manager. The results include the data, the evaluation of uncertainty, and the "most likely" case. All tables and figures will follow the written explanation.

The first six pages of the results would include the data and will be identical to Tables 3, 4, 5, 6, 7, and 8, in Chapter II. Table 3, the first page of the results, would typically include a case identification if the method was standardized with a computer program. The case identification would include whatever details the user would want to include about the uranium mine in New Mexico and the uncertainty analysis.

Following the data, the results of the uncertainty analysis are presented. In terms of interpretation, it should be emphasized at this point that all uncertainty results are consistent with the data the five panel experts provided, but not necessarily accurate. The accuracy of the uncertainty analysis will depend on the accuracy of the experts' forecasts. Consistency and accuracy will be discussed in more detail in the Critical Evaluation, Section V.E. For now, the manager should note that for each proposed project evaluated by this method, statistics are calculated that estimate the mean or expected AARR, the variance of AARR (a measure of risk), the "most likely" AARR, and so on. These estimates reflect the forecasts of the panel of experts. This information, although not perfectly accurate, is the best information available at that point in time and should be very useful in distinguishing which projects offer greater returns and less risk.

The results of the uncertainty analysis begin with a summary, Figure 5. Several of the statistics in the summary are taken from the frequency table, Table 17. The frequency table and summary are self-explanatory. Also from the frequency table, a frequency distribution on AARR, Figure 6, is formed, which estimates the probability distribution function on AARR. Figure 7 graphically represents the last column of the frequency table.

From the matrix of AARR's, Figure 4, two sensitivity-type graphs are plotted. Figure 8 shows the expected AARR at each of the ten price schedules in Tables 13 and 14. The 1981 prices are measured on the abscissa and the mean AARR, found in the last column of Figure 4, are then plotted at each price. If an individual evaluator believes prices do not present significant uncertainty, yet recognizes that variable operating costs do, that manager can identify a 1981 price that approximates his/her estimate and then, provided the average escalation rates for price are acceptable, find the expected AARR at that price. The graph also shows the sensitivity of the proposed project to price changes.

Figure 9 shows the expected AARR at each of the ten variable operating cost schedules in Tables 15 and 16. This graph can be used in similar fashion as Figure 8.

The next two pages of the results would be identical to

The following statistics were computed using forecasts made by a panel of experts:

E(AARR) = MEAN AARR = 18.18%	P(AARR > 5%) = 97%
"MOST LIKELY" AARR = 17.8%	P(AARR > 10%) = 89%
VARIANCE (AARR) = 39.83	P(AARR > 15%) = 72%
STD. DEVIATION (AARR) = 6.31	P(AARR > 20%) = 43%

The uncertain variables and the number of experts providing forecasts for each are:

PRICE ($/ton)	5 Experts
VARIABLE OPERATING COSTS ($/ton)	5 Experts

The uncertain variables were assumed to be statistically independent. Correlation coefficients (r_i) and test statistics, t_{c_i}, are calculated for each pair of uncertain variables. If $|t_c| < t_{.975}$, (n-2) degrees of freedom (taken from a student's t table), then the variables are sufficiently independent. Independence is doubtful if $|t_c| > t_{.975}$, (n-2) d.f.

Variables: PRICE AND VARIABLE OPERATING COSTS;

$r = .662 \qquad t_{c_1} = 1.530 \qquad n = 5$

Figure 5. Summary of uncertainty analysis

Table 17. AARR frequency table

AARR ($AARR_L$ – $AARR_U$)	Frequency	Cumulative Frequency (C.F.), % =	P(AARR $AARR_U$) (100-C.F.),%
0 ≤ AARR < 1	1	1	99
1 ≤ AARR < 2	1	2	98
2 ≤ AARR < 3	0	2	98
3 ≤ AARR < 4	1	3	97
4 ≤ AARR < 5	0	3	97
5 ≤ AARR < 6	2	5	95
6 ≤ AARR < 7	1	6	94
7 ≤ AARR < 8	1	7	93
8 ≤ AARR < 9	3	10	90
9 ≤ AARR <10	1	11	89
10 ≤ AARR <11	2	13	87
11 ≤ AARR <12	4	17	83
12 ≤ AARR <13	3	20	80
13 ≤ AARR <14	4	24	76
14 ≤ AARR <15	4	28	72
15 ≤ AARR <16	5	33	67
16 ≤ AARR <17	6	39	61
17 ≤ AARR <18	6	45	55
18 ≤ AARR <19	6	51	49
19 ≤ AARR <20	6	57	43
20 ≤ AARR <21	7	64	36
21 ≤ AARR <22	6	70	30
22 ≤ AARR <23	6	76	24
23 ≤ AARR <24	5	81	19
24 ≤ AARR <25	5	86	14
25 ≤ AARR <26	4	90	10
26 ≤ AARR <27	3	93	7
27 ≤ AARR <28	4	97	3
28 ≤ AARR <29	2	99	1
29 ≤ AARR <30	1	100	0
TOTAL	100		

E(AARR) = MEAN AARR = 18.18%

VARIANCE(AARR) = 39.83

```
                                        X
                                X X X X X X X
                              X X X X X X X X X X
                      X   X X X X X X X X X X X X X   X
                X     X X X X X X X X X X X X X X X X X
          X     X   X X X X X X X X X X X X X X X X X X X
X X   X   X X X X X X X X X X X X X X X X X X X X X X X X X    AARR (%)
0         5         10        15        20        25        30
```

Figure 6. Frequency distribution on AARR

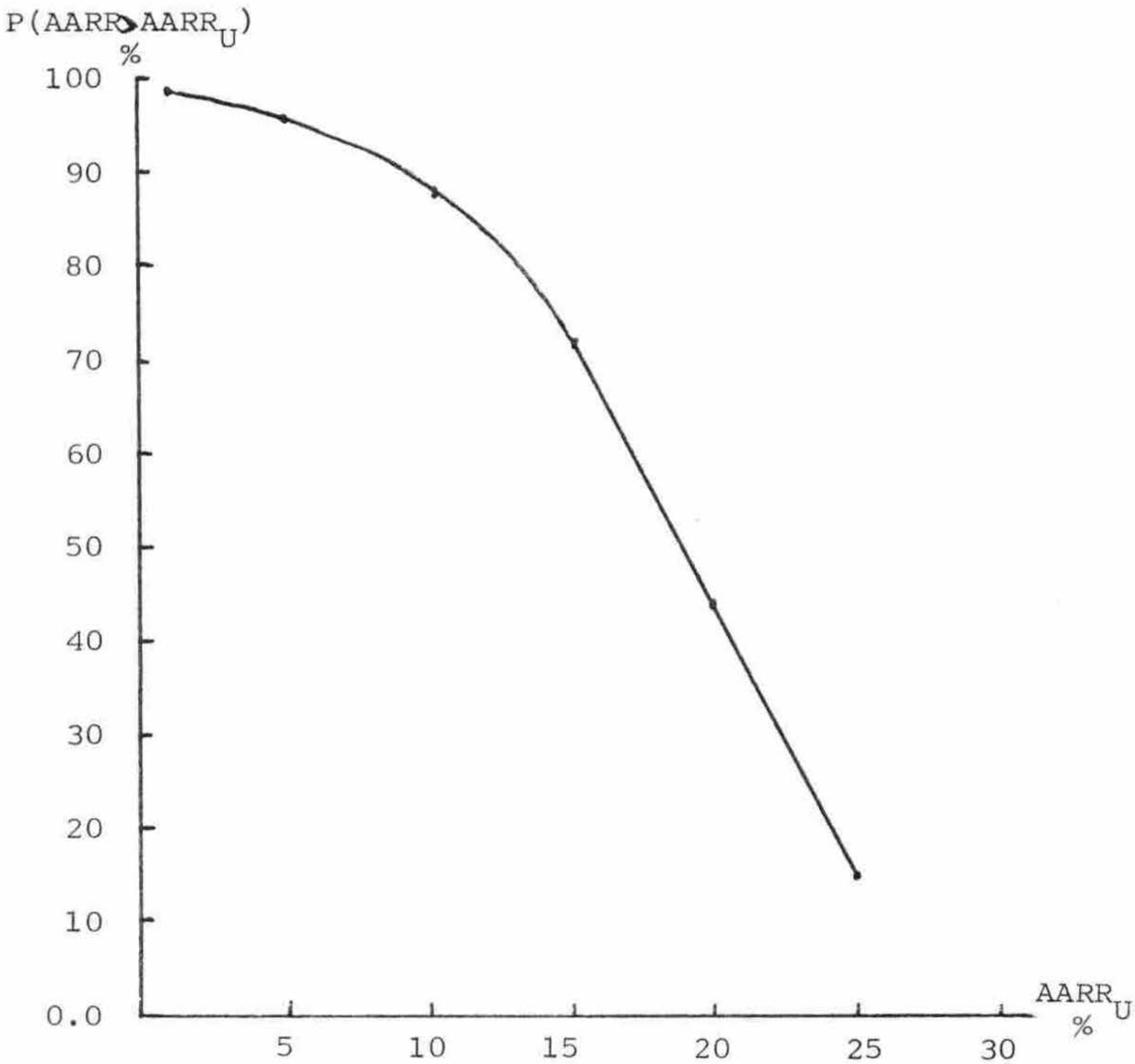

Figure 7. Probability that AARR will be greater than $AARR_U$

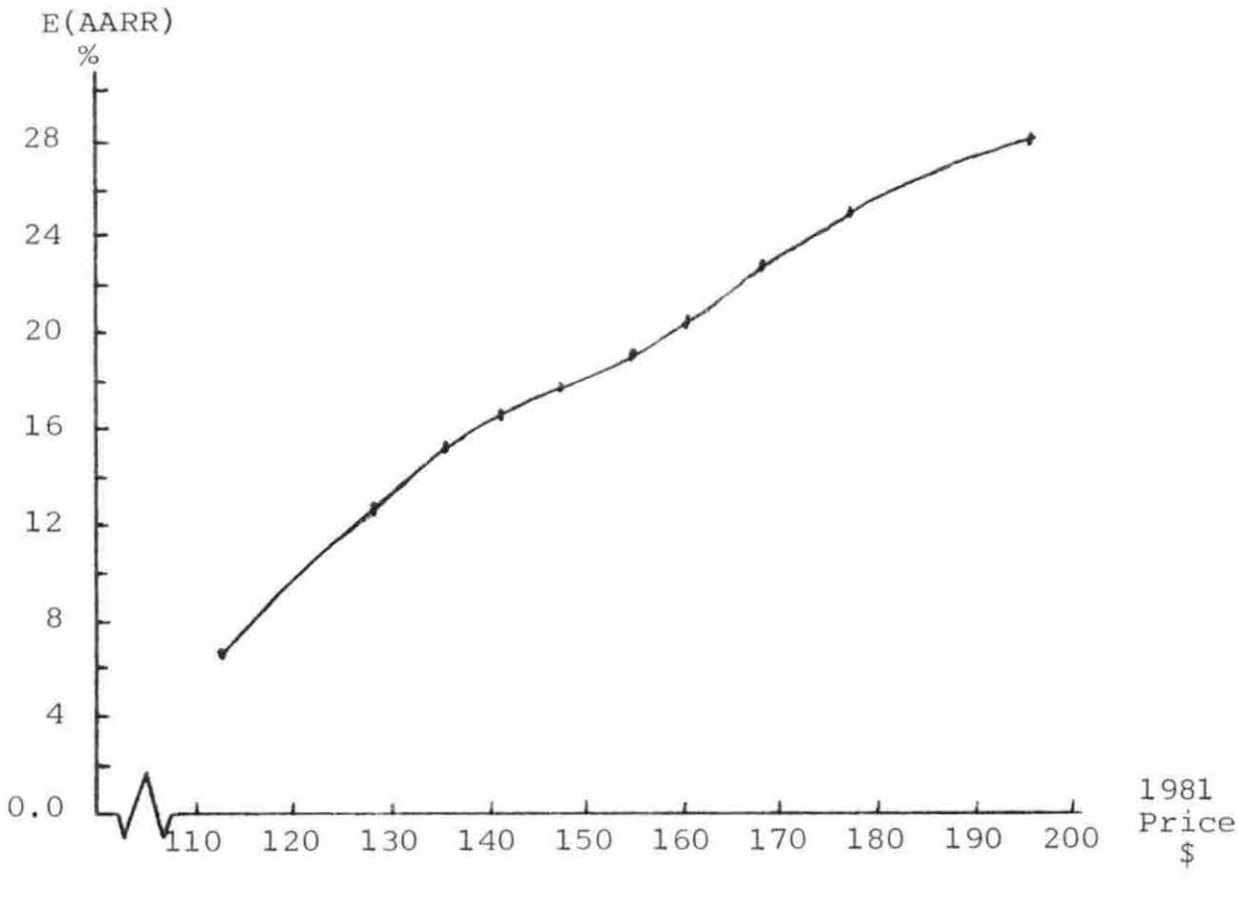

Figure 8. Expected AARR for ten price schedules (assuming escalation is as in Table 4 for years 1982-1994)

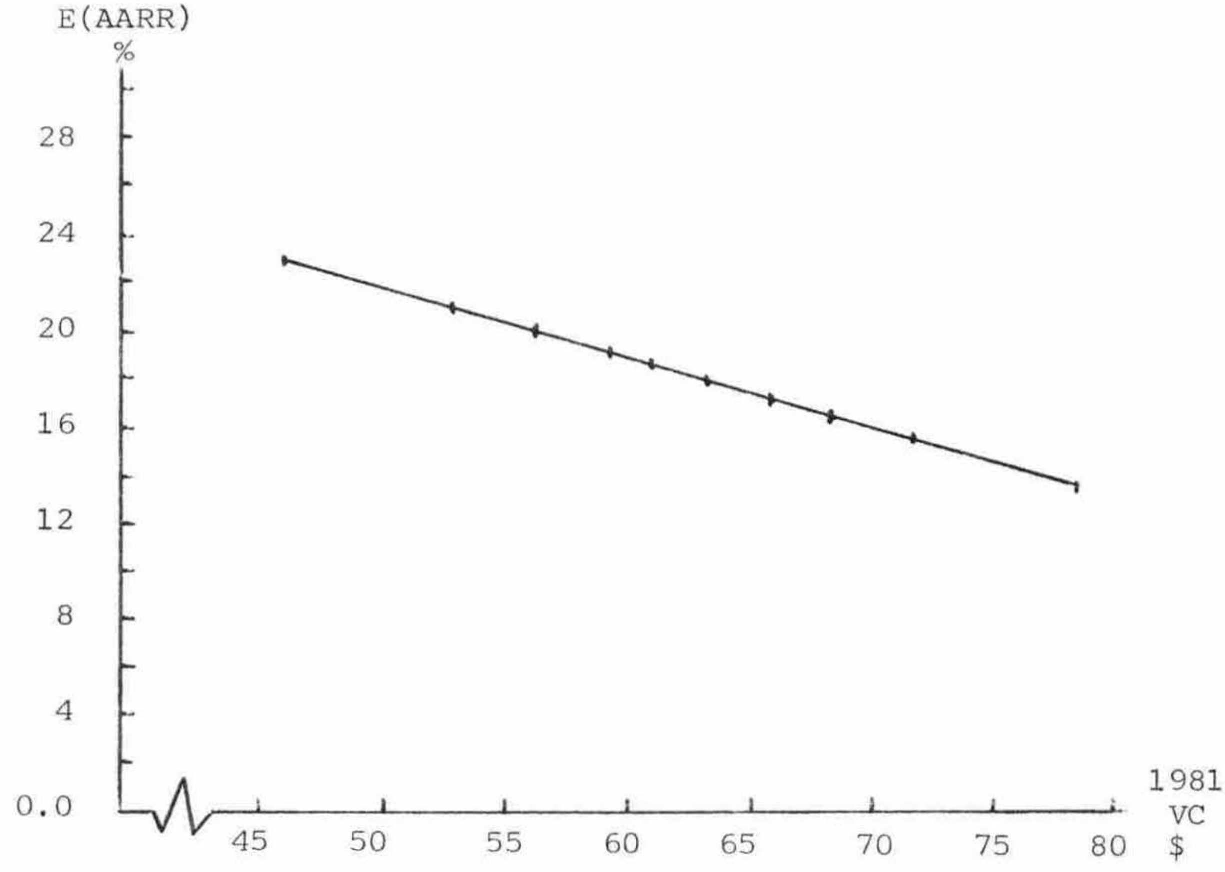

Figure 9. Expected AARR for ten variable operating cost schedules (assuming escalation is as in Table 5 for years 1982-1994)

Tables 13 and 14, and show the ten "possible" price schedules. Each schedule was assigned a probability of occurrence equal to one-tenth.

Tables identical to Tables 15 and 16 would follow, showing the ten "possible" variable operating cost schedules. Each of these schedules were also assigned a probability of occurrence equal to one-tenth.

Lastly, the "most likely" case would be shown in detail so that the evaluators can observe individual characteristics the project is likely to yield, for instance, cash flow, depletion allowances, and so on. The detail of the most likely case is shown in Tables 9, 10, and 11.

E. Critical Evaluation

1. Introduction

The critical evaluation will be conducted in three parts. The advantages of the statistical method will be discussed first, pointing out the benefits of the consistent results to the manager and the ease of conducting the analysis if computerized. The shortcomings of the method will then be presented, concentrating on the assumptions listed in Section V.C. Lastly, extensions to this method and further work to be done with respect to the assumptions will be discussed.

2. Virtues of the statistical method

The benefits of the statistical procedure to both the individual project evaluator and the manager will be discussed in this section.

Individual evaluators are given the opportunity to contribute their assessment or forecast of what the future will be with respect to the uncertain variables. The individual evaluator remains anonymous and, therefore, is free to make his/her forecasts without justification. In this way the forecasts will reflect both empirical research and "expert intuition".

After the team of project evaluators (or experts) has conducted the uncertainty analysis, the manager is supplied with results which are consistent with the data provided by the experts. Concentrating first on this problem, the concept of consistency becomes more clear as several situations or deviations from this case are considered.

First, if one of the experts would have provided a higher maximum and/or higher minimum price for 1981, then the average maximum and/or minimum prices would have been higher, and therefore, the range and shape of the distribution on AARR would have been different, yielding in general higher AARR's. This is consistent with what would be desired. If any of the experts would have been more optimistic, then the uncertainty evaluation should reflect it.

Another case to consider would be if one of the experts would have been more optimistic about only the most likely price. One greater "most likely" price forecast for 1981 would have increased the average "most likely" price and shifted the ten 1981 prices in Tables 24 and 25 to the right. Higher 1981 prices would result in higher AARR's. Again, this is consistent with what is desired or expected.

A third case would be if one expert would have been more pessimistic with respect to inflation. If the expert would have provided higher escalation rates for either prices or variable operating costs, the ranges of the ten prices or costs from 1982 through 1994 would be greater and consequently, the range and distribution on AARR would be more dispersed indicating more risk, which is consistent with the fact that higher inflation rates are associated with greater risk.

To illustrate the last situation, consider the following simplified and exaggerated case. Suppose the original average 1981 minimum and maximum prices are \$10 and \$100, respectively, and the escalation rates progress such that the 1994 minimum and maximum prices are \$20 and \$200, respectively. In this case, from Equation 3 in Section II.C,

$$\prod_{i=1982}^{1994} \left\{1 + \left[\begin{pmatrix}\text{Average}\\ \text{Escalation}\\ \text{Rate, \%}\end{pmatrix}_i \times .01\right]\right\} = 2.$$

Now, if one expert would have provided higher escalation (inflation) rates, then the average escalation rates for each year would have been higher. For illustration, let

$$\prod_{i=1982}^{1994} \{1 + [(\text{Average Escalation Rate, \%})_i \times .01]\} = 5.$$

Now the 1994 range is \$50 to \$500; the minimum price has increased by only \$30 while the maximum price has increased by \$300. Clearly, with prices increasing more rapidly from 1982 through 1994 and finally reaching the \$500 neighborhood, the distribution on AARR would be much more dispersed. The lower AARR's would be changing due to a \$30 increase in price while the upper AARR's would be reflecting a \$300 increase in price.

Turning now to consistency from one evaluation to another, note that each case will yield different frequency distributions for AARR. The distributions will each be determined by the average minimum, maximum, and most likely values for the uncertain variables in this first year and the average escalation rates for each year thereafter. Note that the narrower the range of the average minimum and maximum values of the uncertain variables, the narrower the range of AARR would be (less risk). Therefore, proposed projects can be evaluated and compared on a risk and profit potential basis.

On a final note, with respect to the results of the statistical procedure, the author would like to point out that, although the method was designed for consistency and therefore supported on that basis thusfar, the method may very well prove to be fairly accurate. The accuracy of the results will depend on the accuracy of the experts' forecasts, particularly the "most likely" forecasts. These forecasts, since they reflect a good deal of experience, research, and business intuition, could very possibly be the best information available, and therefore, when used with the procedure described in this paper, could also yield good estimates of the probability distribution function on AARR for each proposed project.

In terms of the ease of conducting evaluations of uncertainty, the method presented in this paper would be especially suitable for computer programming. Once programmed and available on line as cash flow programs are now, evaluations could be conducted rapidly and easily. The ease of evaluating each project would be conducive to evaluating and comparing all projects on a risk basis and should, therefore, improve the decision-making process of the operating group. The manager(s) would still be faced with the risk-return trade-off decision, but clearly, more information is preferred to less.

3. Shortcomings of the statistical procedure

In discussing the shortcomings of the method, the focus will be on the four assumptions made in Section V.C.

The first assumption deals with the possibility of evaluators deliberately providing biased data to enhance the risk and/or profit profile of the project. Although this assumption is significant in that biased data would discredit the entire evaluation, it is an assumption that is always implicitly present when conducting economic feasibility studies. The assumption is included in this paper due to the nature of the data gathering. It would be very easy and tempting for the experts to collaborate prior to being physically separated to make the forecasts.

The second assumption is more critical. It states that the uncertain variables are assumed to be statistically independent with one another and the determinate data. The concept of independence is not a difficult one and therefore, ultimately, the experts are responsible for evaluating only independent variables.

The method provides for an aid in detecting patterns the experts portray in the relationships between the "most likely" values of the uncertain variables. If two variables are dependent in a linear fashion, that pattern will likely be exposed in the forecasts made by the experts, and subsequently be detected by the correlation coefficient between the two variables. Using the procedure described in the Summary (Figure 5), the null hypothesis that $\rho=0$ may or may not be rejected. If the test fails to reject $\rho=0$, then this is considered to be a sufficient condition for independence, but not a necessary one. Even if there is a perfect pattern ($r=1$), one might approach, for instance, the two experts making the pair-wise smallest and largest forecasts and suggest that a dependency exists only to have those experts agree that each of them could be right with respect to only one variable, for example, the minimum price and maximum cost could occur or the maximum price and minimum cost could occur.

Still, the author recognizes that strong dependencies are often present and may go undetected. Suggestions for further work in this area are discussed in the next section.

On a critical note, if the variables are strongly dependent, then the results of the evaluation will be strongly biased since many of the combinations used to calculate AARR's are not actual possible combinations. Therefore, the evaluators are encouraged to be especially aware of this assumption and make an effort to reduce the uncertain variables to only those that are independent. Those that are dependent should be incorporated in the cash flow program accordingly.

The independence of the uncertain variables to the determinate data presents a lesser problem. Most cash flow programs are constructed to make calculations according to dependencies, as was the one used in this problem. Although many variables vary directly with others, they can be programmed accordingly, and the rates still be quite independent. These rates include production tax rates, days accounts receivable, royalty payment (% of mining revenue), and so on.

The third assumption is not so critical. It states that there is no uncertainty with respect to the determinate data. Many variables can be predicted with a good deal of certainty and when some are predicted slightly inaccurately, it is very possible that they could be offsetting with respect to the result of the AARR calculation.

The fourth assumption is very bold and is a major source of criticism. It states there is no uncertainty with respect to relative rates of future escalation (inflation). The main criticism is not that there is only one schedule of escalation rates used for each variable. One schedule for each was chosen because of the recognized dependency between the two schedules. The two schedules may be different, but a "minimum" schedule for prices is not combined with a "maximum" schedule for costs because each pair of escalation rates, in actuality, are subject to similar inflationary pressure each year.

The main criticism is that there is no allowance made to reflect greater uncertainty or risk when the forecasted escalation rates are more dispersed among the experts in one case as compared to another. For instance, the average rates of two projects might be identical, yet if there is little disagreement among the experts in one project and a great deal in the other, more risk should be associated with the latter. A measurement of that additional risk is not included at present.

4. Extensions to the procedure and further work

Several extensions and modifications to the statistical procedure are recommended in the event a computer program is written to perform the uncertainty analysis.

First, provisions should be made to accommodate more than five experts, say for instance, as many as 20. Also, more uncertain variables should be allowed. The evaluator might want to be able to select, for instance, four variables. Typical variables would include prices, variable operating costs, production schedules, investment costs, and so on.

A significant modification to a programmed procedure, in the interest of saving computer time, would be to randomly select a fraction of the one hundred possible combinations, say thirty, to construct the frequency distribution and compute the statistics. The savings in computer time would be well worth the small loss of accuracy, particularly if more than two uncertain variables are being analyzed, in which case the possible number of combinations might be a thousand or million, depending on the number of intervals the analyst chooses for segmenting the uncertain variables' triangular distributions, which could very well be different than ten and is up to the analyst's discretion.

With respect to the second assumption in Section V.C that states that the uncertain variables are statistically independent, the author recommends further work investigating the historical relationship between the uncertain variables chosen for evaluation. In the case of this problem, paired observations could be obtained from similar mining projects already underway that might reveal a pattern. Standard econometric tools could be used to estimate linear relationships among these two and/or other variables. If, for instance, a linear model can be estimated that explains a large proportion of the variation, say ninety percent, the flexibility to use that linear relationship could be made available in the cash flow program.

Lastly, if a computer programmed procedure is developed, it should be programmed to accommodate or "link" more than one cash flow program. Many cash flow programs are typically available in large corporations that perform varied functions, for example, some cash flow programs allow for equity financing, others can handle tax structures of foreign countries, and so on. It should be an option in the program to direct the determinate data and differing combinations to the cash flow program of the user's choice. In this way, different shops now using different cash flow programs in their evaluation process could all use the same uncertainty evaluation program by simply selecting their program among the optional cash flow programs. The details of coordinating the formats of the input data to the optional cash flow programs is left to the computer analyst and programmer.

VI. CONCLUSION

Using price and cost forecasts made by five mining experts, a statistical procedure was described that produced a frequency distribution on AARR. Since the procedure is very suitable for computer programming, uncertainty analysis for all proposed projects could be quickly and easily conducted to provide managers with an estimated risk assessment for each project. The additional information would improve the company's evaluation and project selection process since at present only point estimates of AARR are used.

The frequency distribution estimates the actual probability distribution function on AARR. Therefore, statistics computed from the frequency distribution are also estimates.

With respect to this problem, several useful statistics were calculated. The expected AARR was 18.18%, the variance (AARR) was 39.83, and the probability of attaining an AARR greater than 15% was 72%.

The paper was written such that an economic computer analyst would have the necessary information to design a computer program that would facilitate the uncertainty analysis. Also included are three sections for the manager that describe the data gathering procedure, the results, and a critical evaluation of the method.

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IX. APPENDIX A: CASH FLOW PROGRAM

```
$JOB 'X',TIME=6,PAGES=25
C
C     CASH FLOW PROGRAM TO CALCULATE 5 AARR'S; 1 PRICE AND 5 COSTS
C
      DIMENSION DINV(23), DEVCST(23),RLSHD(23), EXPL(23), WK(23),
     1DEXP(23), PTX(23), FIN(23), STX(23), ADTX(23), DRE(23), DEP(23),
     2RITC(23),DINC(23), P(23), C(23), TXRT(23), GREV(23), VCST(23),
     3TWK(23), RDYEXP(23), FOVHD(23), TPTX(23), TOTEXP(23), IBFIT(23),
     4TXDDCT(23), DEPL(23), TXINC(23), NETTX(23), IAFIT(23), CASH(23),
     5TINV(23), CF(23), DISCF(23)
C
C     READ DATA IN TABLES 6-8
C
      DO 10 I=1,23
      READ(5,5) DINV(I), DEVCST(I), RLSHD(I), EXPL(I), WK(I)
    5 FORMAT(5F10.0)
   10 CONTINUE
      DO 15 I=1,23
      READ(5,5) DEXP(I), PTX(I), FIN(I), STX(I), ADTX(I)
   15 CONTINUE
      DO 25 I=1,23
      READ(5,5) DRE(I), DEP(I),RITC(I), TXRT(I), DINC(I)
   25 CONTINUE
C
C     READ A PRICE SCHEDULE
C
      DO 35 I=1,23
      READ(5,30) P(I)
   30 FORMAT(F10.0)
   35 CONTINUE
C
C     FIVE AARR'S WILL BE CALCULATED
C
      DO 999 ICOST=1,5
C
C     READ A COST SCHEDULE
C
      DO 40 I=1,23
      READ(5,30) C(I)
   40 CONTINUE
      TRWK = 0.0
      TLSHD=0.0
      TDRE=0.0
      TDPL=0.0
      TEXPL=0.0
      DO 45 I=1,23
      TEXPL=TEXPL + EXPL(I)
      TDRE=TDRE + DRE(I)
```

```
      TLSHD=TLSHD +RLSHD(I)
      DEPL(I) = 0.0
   45 CONTINUE
C
C     LOOP TO CALCULATE CASH FLOW
C
      DO 500  J=1,23
      GREV(J) = ORE(J)  * P(J)
      VCST(J) = ORE(J)  * C(J)
      IF  (J.EQ.1)GO  TO 50
      TWK(J)=WK(J)+((GREV(J)-GREV(J-1))*60/365)+((VCST(J)-VCST(J-1))*
     740/365)
   50 CONTINUE
      TWK(1)  = 0.0
      ROYEXP(J)=GREV(J)  * 0.0527
      FOVHD(J)=FIN(J) +  (TWK(J)  * 0.0165)
      TPTX(J)=PTX(J)  +  ((GREV(J)-ROYEXP(J))  * 0.01089)
      TOTEXP(J)=OEXP(J)  +  STX(J)+ADTX(J)+VCST(J)+ROYEXP(J)+FOVHD(J)+
     8TPTX(J)
      IBFIT(J)=GREV(J)-TOTEXP(J)
      TXDDCT(J)=EXPL(J)+DEVCST(J)+DEP(J)
C
      IF(J.LT.9)GO TO 90
C
C     CALCULATE COST DEPLETION
C
      IF(TLSHD.LT.TDPL)GO  TO 60
      CDPL=(ORE(J)/TORE)  *  (TLSHD-TDPL)
      TORE  =  TORE  -  ORE(J)
      GO  TO 70
   60 CDPL=0.0
      GO  TO 70
   70 CONTINUE
C
C     CALCULATE DEPLETION, % OF GROSS
C
      DPLG=(GREV(J)-ROYEXP(J))  *  .22
C
C     CALCULATE DEPLETION, % OF NET
C
      DOVHD=((GREV(J)-ROYEXP(J))  *  .065)  +  (DEVCST(J)  *  .155)
      DPLN=(IBFIT(J)  +  FOVHD(J)  -  DOVHD  -  DEP(J)  -  DEVCST(J))  *  .50
C
C     DETERMINE DEPLETION 'ALLOWANCE'
C
      DPL1 = DPLG
      IF(DPLN.LT.DPLG)DPL1  =  DPLN
      IF(CDPL.GT.DPL1)DPL1  =  CDPL
      TDPL  =  TDPL  +  DPL1
```

```
C
C     RECOVER EXPLORATION COSTS FROM DEPLETION
C
      DEPL(J) = DPL1 - TEXPL
      IF(DEPL(J).LT.0.0)GO TO 80
      TEXPL = 0.0
      GO TO 90
   80 TEXPL = -DEPL(J)
      DEPL(J) = 0.0
      GO TO 90
C
C     CALCULATE TAXABLE INCOME AND CASH FLOW
C
   90 CONTINUE
      TXINC(J) = IBFIT(J) - TXDDCT(J) - DEPL(J)
      NETTX(J) = (TXINC(J) * TXRT(J)) -RITC(J)
      IAFIT(J) = IBFIT(J) - NETTX(J)
      CASH(J) = IAFIT(J) + DINC(J)
      IF(J.LT.23)GO TO 100
      DO 95 I=1,23
      TRWK = TRWK + TWK(I)
   95 CONTINUE
      CASH(J) = IAFIT(J) + TRWK + 5836351.0
  100 CONTINUE
      TINV(J) = DINV(J) + DEVCST(J) +RLSHD(J) + EXPL(J) + TWK(J)
      CF(J) = CASH(J) - TINV(J)
  500 CONTINUE
C
C     CALCULATE AARR
C
      RLOW=0.0
      RHIGH=1.0
      N=0
   20 R=(RLOW + RHIGH)/2
      N=N+1
      IF(N.GE.50) GO TO 550
      DISCF(1)=CF(1)
      DO 525 I=2,23
      K=I-2
      DISCF(I)=CF(I)/(((1.0 + R)**K) * (1.0 + (R/2)))
  525 CONTINUE
      SUM= 0.0
      DO 530 K=1,23
      SUM = SUM + DISCF(K)
  530 CONTINUE
      IF(ABS(SUM).LT.100.0) GO TO 550
      IF(R.LT.0.001) GO TO 550
      IF(SUM)535,550,540
  535 RHIGH=R
```

```
      GO TO 20
  540 RLOW=R
      GO TO 20
C
C     PRINT RESULTS
C
  550 WRITE(6,600)
  600 FORMAT('1',5X,'PRICE',6X,'VAR COST',2X,'GROSS REV',2X,'TOTAL VC',
     +4X,'ROY EXP',1X,'TOT FOVHD',2X,'TOT PTAX')
      DO 620 I=1,23
      WRITE(6,610) P(I), C(I),GREV(I), VCST(I), ROYEXP(I), FOVHD(I),
     +TPTX(I)
  610 FORMAT(1X,2(2X,F10.2),5(2X,F10.0))
  620 CONTINUE
      WRITE(6,630)
  630 FORMAT('0',3X,'OTHER EXP',3X,'SALES TAX',3X,'AD VAL TAX', 3X,
     +'TOT EXP',5X,'IBFIT',5X,'TAX DDUCT',6X,'DEPL')
      DO 640 I=1,23
      WRITE(6,635) OEXP(I), STX(I), ADTX(I), TOTEXP(I), IBFIT(I),
     +TXDDCT(I), DEPL(I)
  635 FORMAT(1X,4(2X,F10.0),2X,I10,2(2X,F10.0))
  640 CONTINUE
      WRITE(6,650)
  650 FORMAT('1',5X,'TAX INC',5X,'NET TAX',6X,'IAFIT',8X,'CASH',6X,
     +'TOT WK',4X,'TOT INV',5X,'CASH FLOW',4X,'DISC CF')
      DO 660 I=1,23
      WRITE(6,655) TXINC(I), NETTX(I), IAFIT(I), CASH(I), TWK(I),
     +TINV(I),CF(I), DISCF(I)
  655 FORMAT(3X,F10.0,2(2X,I10),5(2X,F10.0))
  660 CONTINUE
      AARR=R * 100.
      WRITE(6,800) AARR
  800 FORMAT('0',1X,'AARR = ',F8.5,1X,'%')
  999 CONTINUE
      STOP
      END

$ENTRY
```

PRICE	VAR COST	GROSS REV	TOTAL VC	ROY EXP	TOT FOVHD	TOT PTAX
0.00	0.00	0.	0.	0.	0.	0.
0.00	0.00	0.	0.	0.	0.	0.
0.00	0.00	0.	0.	0.	60819.	0.
0.00	0.00	0.	0.	0.	161782.	0.
0.00	0.00	0.	0.	0.	457726.	0.
0.00	0.00	0.	0.	0.	801104.	0.
0.00	0.00	0.	0.	0.	1415619.	0.
0.00	0.00	0.	0.	0.	1932954.	0.
146.31	61.60	59255530.	24948000.	3122766.	2392097.	4722035.
154.21	66.47	62455040.	26920330.	3291380.	2487354.	4755042.
162.54	70.59	103537900.	44965820.	5456451.	2687217.	7533657.
172.05	74.68	126800800.	55039150.	6682404.	2822898.	8788639.
182.03	78.86	134156000.	58119800.	7070026.	2912092.	8864517.
191.49	83.20	141128100.	61318380.	7437452.	3005319.	8936441.
201.45	87.47	148468600.	64465370.	7824297.	3066440.	9012166.
204.67	90.85	150841700.	66956440.	7949361.	3082644.	9036648.
204.67	91.81	150841700.	67663960.	7949361.	3086089.	9036648.
204.67	91.81	150841700.	67663960.	7949361.	3084810.	9036648.
204.67	91.81	150841700.	67663960.	7949361.	3084810.	9036648.
204.67	91.81	150341700.	67663960.	7949361.	3084810.	9036648.
204.67	91.81	150841700.	67663960.	7949361.	3084810.	9036648.
204.67	91.81	124030000.	55636840.	6536382.	2990340.	7430405.
0.00	0.00	0.	0.	0.	2552057.	0.

OTHER EXP	SALES TAX	AD VAL TAX	TOT EXP	IBFIT	TAX DDUCT	DEPL
44.	0.	0.	44.	-44	10640.	0.
44.	0.	0.	44.	-44	98448.	0.
440.	0.	0.	61259.	-61259	3893492.	0.
236440.	16427.	7529.	422178.	-422178	6045000.	0.
1089024.	528386.	247700.	2322836.	-2322836	5498397.	0.
1464024.	646407.	532109.	3443643.	-3443643	9437145.	0.
2722024.	1179330.	1041581.	6358554.	-6358554	15339610.	0.
4112860.	946706.	1443959.	8436479.	-8436479	4330466.	0.
-3666765.	647047.	1725038.	33890190.	25365344	15872070.	3434568.
476034.	163006.	1799747.	39892880.	22562160	13120250.	3840786.
554030.	108423.	1849441.	63155020.	40382944	11423680.	12463760.
611189.	105480.	1897786.	75947520.	50853328	9856770.	17875020.
579192.	123717.	1954490.	79623800.	54532288	9183138.	19861230.
483836.	132713.	2015317.	83329420.	57798704	8699146.	21558650.
522332.	67419.	2046217.	87004220.	61464416	6395278.	24483000.
543119.	5993.	2048964.	89623150.	61218624	5408184.	24788190.
543119.	0.	2048964.	90328120.	60513648	4395466.	24958110.
543119.	0.	2048964.	90326840.	60514928	3704497.	25303600.
543119.	0.	2048964.	90326840.	60514928	3126496.	25592600.
543119.	0.	2048964.	90326840.	60514928	2642062.	25834810.
543119.	0.	2048964.	90326840.	60514928	2235313.	26038190.
543119.	0.	2048964.	75186030.	48843984	6847121.	18675040.
5002640.	0.	2048964.	9603661.	-9603661	0.	0.

TAX INC	NET TAX	IAFIT	CASH
-10684.	-5128	5084	5084.
-98492.	-47276	47232	47232.
-3954751.	-1898280	1837021	1837021.
-6467178.	-3104245	2682067	2492067.
-7821233.	-3754191	1431355	-568645.
-12880780.	-6182778	2739135	2739135.
-21698160.	-10415117	4056563	4056563.
-12766940.	-6128133	-2308346	-2308346.
6058704.	-4009096	29374440	29374430.
5601123.	2644529	19917631	19917610.
16495480.	8314426	32068518	32068510.
23121520.	11655614	39197714	39197710.
25487900.	12822750	41709538	41709530.
27540890.	13856570	43942134	43942120.
30586120.	15433403	46031013	46031000.
31022240.	15821342	45397282	45397280.
31160060.	15891632	44622016	44622010.
31506810.	16068475	44446453	44446440.
31795820.	16215869	44299059	44299050.
32038040.	16339404	44175524	44175520.
32241400.	16443117	44071811	44071800.
23321800.	11894121	36949863	36949850.
-9603661.	-4897867	-4705794	2417341.

AARR = 17.80185 %

TOT WK	TOT INV	CASH FLOW	DISC CF
0.	16070.	-10986.	-10986.
0.	101648.	-54416.	-49968.
0.	3896692.	-2059671.	-1605514.
0.	6177200.	-3685133.	-2438470.
10863.	17935980.	-18504620.	-10394230.
158657.	20810810.	-18071680.	-8617049.
435576.	37243420.	-33186840.	-13433020.
4257470.	31353680.	-33662010.	-11566330.
8898937.	27826800.	1547632.	451410.
742092.	5773149.	14144460.	3502176.
8730944.	12077320.	19991180.	4201823.
4927960.	8183520.	31014190.	5533597.
1546687.	5365123.	36344400.	5504685.
1496615.	5592703.	38349420.	4930621.
1551534.	3632365.	42398640.	4627458.
663099.	848075.	44549200.	4127418.
77536.	77536.	44544480.	3503323.
0.	0.	44446440.	2967367.
0.	0.	44299050.	2510596.
0.	0.	44175520.	2125260.
0.	0.	44071800.	1799860.
-5725452.	-5725452.	42675290.	1479457.
-26485660.	-26485660.	28902990.	850583.

X. APPENDIX B: CASH FLOW EQUATIONS

The cash flow equations (4-22) presented in Chapter II will now be described in more detail.

The Gross Revenue and Variable Operating Cost equations (4 and 5) are straightforward. Prices ($/ton) are for the particular grade of uranium ore expected to be mined. Variable Operating Costs ($/ton) include expenses that occur on a production basis, such as labor costs and fuel costs. These types of costs and prices contribute a significant amount of uncertainty to the evaluation.

Total Working Capital requirements, Equation 6, include investment costs which occur due to the timing of payments and receipts. For instance, in the gross revenue calculation, receipts are included for a full year's production and delivery, however, 60 days of those receivables are not expected to be received until the next year. Therefore, 60/365 of that year's receivables are an investment cost in that year. Those 60/365 receivables will be received in the next year. So if production and prices remain constant, no additional working capital would be needed throughout the project with respect to receivables. The flow of revenue would simply be delayed 60 days and in the year after production ends there would be an additional 60 days of revenue (a return of working capital). But since prices and production change, working capital is calculated as 60/365 of the increment in gross revenue each year, even if that increment is negative.

Similarly, "days product inventories" is a working capital requirement since inventories are kept at a percentage (40/365) of the year's production. This portion of the working capital is calculated on the yearly increment of variable operating cost because variable operating costs are closely associated to the cost of inventories.

The working capital from Table 6 is given and constant for all data cases.

Royalty Expense, Equation 7, includes payments to the lessor as a percent of gross revenue, contracted by the lessor and lessee.

Total Financial Overhead, Equation 8, is explained satisfactorily in Section II.E.

Total Production Taxes, Equation 9, are taxes paid to state government and are computed as a percentage of the revenue from the mine plus a dollar per ton charge. The dollar per ton charges are the production taxes from Table 7 ($10.15/ton).

Total Expense, Equation 10, is the sum of seven expense categories. Other Expense is given. Sales Taxes are computed on investments and are constant for each case. Ad Valorem Taxes are property taxes, also computed on capital investments. Variable Operating Costs, Royalty Expense, Total Financial

Overhead, and Total Production Taxes have previously been discussed.

Income Before Federal Income Taxes, Equation 11, is income after expenses.

Tax Deductions, Equation 12, include Exploration Costs and Intengible Development Costs in the year they occur plus the yearly depreciation of the depreciable investments in Table 6, using the double-declining balance method.

The depletion deduction is the greater of cost depletion and percentage depletion each year after exploration costs have been "recaptured" by foregoing the depletion deductions until the cumulative depletion allowances equal the cumulative exploration costs. Exploration costs must be recaptured since they are expensed from gross revenue as a "tax deduction".

The Cost Depletion calculation, Equation 13, "charges off" or depreciates a portion of the leasehold investment in the proportion of the ore mined that year to the total expected amount ore to be mined. The calculation is similar to units-of-production depreciation. Computationally, each year the amount of the lease "depleted" is calculated by dividing that year's ore by the <u>remaining</u> ore and multiplying that fraction by the <u>remaining</u> leasehold investment (total leasehold investment minus the sum of all previous depletion deductions).

Depletion, percent of Gross Revenue, Equation 14, is 22% of the revenue occurring to the lessee (or operator). The 22%

rate is for uranium. Other ores are allowed different percentages.

The percentage depletion allowance cannot exceed 50% of net revenue, therefore, Depletion, % of Net Revenue, Equation 15, is calculated. Net revenue is revenue less expenses and the capital consumption (depreciation).

The Depletion Overhead calculation, Equation 16, is stipulated by the Internal Revenue Service, rather than allow each company to use their own overhead calculation.

Percentage depletion is essentially a gift. The percentage depletion allowance is the lesser of percent of gross revenue and percent of net revenue. If the percentage depletion calculated is larger than the cost depletion calculated, then the percentage depletion is the allowable depletion deduction.

The Taxable Income, Net Tax, and Income After Federal Income Taxes equations (17-19) are straightforward. Investment Tax Credits (Table 8) are computed as a percentage of certain qualified investments.

Cash Income, Equation 20, is the sum of Income After FIT, salvage expected from the capital equipment depreciated, outside income (given), and the return of working capital. Return of Working Capital is the sum of the working capital requirements throughout the project, received in the last year of the project. In this year all outstanding accounts receivables, inventories, and so on, are settled.

Total Investment, Equation 21, includes five investment categories. Depreciable investments are just that. Intangible development costs are investments in the mine that cannot be depreciated or salvaged. They include drilling the mine shafts, roadwork, and so on. Leasehold investments are the yearly payments to the lessor for the rights to the ore. Exploration costs are just that. Exploration costs and intangible development costs were included in the tax deduction equation (12) and are essentially expensed since they cannot be salvaged or depreciated.

Finally, Cash Flow, Equation 22, is the cash received each year minus the payments to investment.